**THREE INVESTIGATIONS.**

1(a) Find the coordinates of the points where the line ***y = ½x*** crosses the

 circle given by ***(x – 4)2 + y2 = 8***

Subs ***y = ½x***  into ***(x – 4)2 + y2 = 8 and we get (x – 4)2 + x2 = 8***

 ***4***

 ***x2 – 8x + 16 + x2 = 8***

 ***4***

 ***4x2 – 32x + 64 + x2 = 32***

 ***5x2 – 32x + 32 = 0***

 ***x = 1.24 and 5.16***

***intersection points are (1.24, 0.62) and (5.16, 2.58)***

(b)

A line of the form ***y = mx*** can cross the circle ***(x – 4)2 + y2 = 8***

once or twice or not at all.

Find the value of ***m*** so that the line is a tangent.

Subs ***y = mx***  into ***(x – 4)2 + y2 = 8 and we get (x – 4)2 + m2x2 = 8***

 ***x2 – 8x + 16 + m2x2 = 8***

 ***(1 + m2) x2 – 8x + 8 = 0***

 ***This equation will only have 1 solution so the discriminant = 0***

 ***= 64 – 4×(1 + m2)×8 = 0***

 ***64 – 32 – 32m2 = 0***

 ***32 = 32 m2***

 ***m2 = 1***

***so m could be 1 or – 1***

***There are 2 possible tangents y = x and y = –x***

2(a)

 

The graphs above show the intersection points of ***y = – 6 and y= x + 5***

 ***x***

Find the coordinates of the intersection points algebraically.

 ***x + 5 = –6***

 ***x***

 ***x2 + 5x = – 6***

 ***x2 + 5x + 6 = 0***

***(x + 2)(x + 3) = 0***

***x = –2 and –3 The intersection points are (–2, 3) and (–3, 2)***

(b) Consider the line ***y = mx + 5*** which has a fixed ***y*** intercept at P but a

 variable gradient ***m***.



P

 A B C

Line A crosses the hyperbola twice, Line B is a tangent and Line C does not cross.

Find the value of ***m*** so that ***y = mx + 5*** is a tangent.

***mx + 5 = – 6***

 ***x***

***mx2 + 5x + 6 = 0***

***to be a tangent = 0 so 25 – 24m = 0 giving m = 25***

 ***24***

(c) Write down the range of values for m so that there will be 2 intersections.

  ***> 0 so 25 – 24m > 0 giving 25 > 24m so 25 > m***

 ***24***

(d) Write down the range of values for m so that there will be no intersections.

  ***< 0 so 25 – 24m < 0 giving 25 < 24m so 25 < m***

 ***24***

3(a) The graphs below show ***y = (x – 3)2 and y = x – 3***

 

Calculate algebraically the intersection points.

 ***x2 – 6x + 9 = x – 3***

***x2 – 7x + 12 = 0***

***(x – 3)(x – 4)=0***

***So x = 3 and x = 4***

***The intersection points are:***

***(3, 0) and (4, 1)***

(b) Consider the line ***y = mx – 7*** which has a fixed ***y*** intercept but a variable

 gradient ***m***.

Find the value of ***m*** so that

***y = mx – 7*** is a tangent to the parabola ***y = (x – 3)2***

 ***x2 – 6x + 9 = mx – 7***

***x2 – (m + 6)x + 16 = 0***

***to be a tangent = 0***

***so (m + 6)2 – 64 = 0***

 ***(m + 6) = 64***

 ***m + 6 = ± 8***

***hence m = 2 or m = – 14***



P

***This means there are 2 tangents from P above.***

***The tangents are y = 2x – 7 and y = –14x – 7***

 P

(c) If ***y = mx – k*** is to be a tangent to ***y = (x – k)2*** show that ***m2 + 4km – 4k = 0***

 ***x2 – 2kx + k2 = mx – k***

 ***x2 – (2k + m)x + (k2 + k) = 0***

***For the line to be a tangent, this equation must have 1 solution so its discriminant = 0***

 ***= (2k + m)2 – 4(k2 + k) = 0***

 ***4k2 + 4km + m2 – 4k2 – 4k = 0***

 ***So m2 + 4km – 4k = 0***