**Towards Excellence.**

1.



(a) Find algebraically, the intersection points of the graphs ***y = 8*** and ***y = -x + 6***

 ***x***

***8 = -x + 6***

***x***

***8 = - x2 + 6x***

***x2 – 6x + 8 = 0***

***(x – 2)(x – 4) = 0***

***x = 2 and x = 4***

***y = 4 and y = 2***

***intersection points are (2, 4) and (4, 2)***

(b)(i) If ***y = -x + c*** is to be a tangent to ***y = 8*** find c

 *x*

***8 = -x + c***



***x***

***8 = - x2 + cx***

***x2 – cx + 8 = 0***

***if there is to be 1 solution***

***then the discriminant = 0***

***so c2 – 4 × 8 = 0***

 ***c2 = 32***

 ***c ≈ ± 5.66***

 (ii) Explain fully why there are two values for c.

 ***A line with a gradient of 2 can be a tangent to each half of the hyperbola.***

 ***See diagram above.***

(c) Find all the values of c for which the line ***y = -x + c*** will cross the

 Hyperbola ***y = 8*** at exactly two points.

 ***x***

***Line will cross twice if there are 2 solutions to x2 – cx + 8 = 0 and this occurs if the discriminant is positive so c2 – 4 × 8 > 0***

 ***c2 > 32***

 ***so c < – 5.66 or c > +5.66***

(d) Find all the values of c for which the line ***y = -x + c*** will NOT cross the

 Hyperbola ***y = 8*** at all.

 ***x***

***Line will not cross if there are no solutions to x2 – cx + 8 = 0 and this occurs if the discriminant is negative so c2 < 32***

 ***so - 5.66 < c < +5.66***

2. The graph shown has the equation ***y = 6 + 6***

 ***x***

 If a line ***y = mx*** is to be a tangent to ***y = 6 + 6*** find ***m.***

 ***x***



***mx = 6 + 6***

 ***x***

***mx2 = 6 + 6x***

***mx2 – 6x – 6 = 0***

***there will be only 1 solution if = 0***

***so 36 – 4m(-6) = 0***

***24m = - 36***

 ***m = - 3***

 ***2***

3. The graph below has the equation ***y = -12 + 9***

 ***x***

 If a tangent has a gradient of 2, find the coordinates of the point where the tangent meets the ***x*** axis.

 

***Let tan be y = 2x + c***

***Intersection is found by solving:***

***2x + c = -12 + 9***

 ***x***

***2x2 + cx = -12 + 9x***

***2x2 + x(c – 9) + 12 = 0***

***Only 1 sol if = 0***

***(c – 9)2 - 4×2×12 = 0***

***(c – 9)2 = 96***

 ***c = 9 ± 9.8***

***c = -0.8 or 18.8***

Explain clearly why there are TWO answers!

***There are two tangents because if the gradient of a line is 2 it can be a tangent to each half of the hyperbola as shown on the diagram.***

***The 2 equations are y = 2x – 0.8 and y = 2x + 18.8***

***These cross the x axis when the y value is 0 ie at x = 0.4 and x = -9.4***