**YEAR 12: Rate of Change, Distance, Velocity and Acceleration.**

1. A drop of petrol drops on the surface of

a pond and spreads in a circular shape.

The radius, r cm , at t sec is given by :

r = 5t + 4

(a) What was the initial radius of the drop

of petrol at the instant it hit the water?

(ie find r at t = 0)

(b) Find the radius at t = 3 sec.

(c) Find the **rate** at which the radius is

increasing.( ie dr )

dt

(d) Find an expression for the

circumference C of the petrol in terms

of t. (ie substitute r = 5t + 4 in the

equation C = 2 π r )

(e) Find the rate of increase of the

circumference.( ie dC )

dt

(f) Find an expression for the area A

of the petrol in terms of t.

(ie substitute r = 5t + 4 in the

equation A = π r2 )

(g) Find the rate of increase of the area at

t sec. ( ie dA )

dt

(h) Find the rate of increase of the area at

t = 0 sec

(i) Find the rate of increase of the area at

t = 3 sec

2. A rocket is being launched and for the

first 10 seconds of its flight, its distance,

H, from the ground at t sec, is given by:

H = 2t3  metres

(a) Find H at t = 1 sec

(b) Find H at t = 10 sec

(c) Find the Velocity equation for the

rocket at t sec. ( ie dH = V )

dt

(d) Find the velocity of the rocket at t = 1

(e) Find the velocity of the rocket at t = 10

(f) Find the acceleration equation for the

rocket. ( ie dV = a )

dt

(g) Find the acceleration at t = 1 sec

(h) Find the acceleration at t = 10 sec

3. A ball is kicked vertically upwards and its

height H at t sec is given by H = 40t – 5t2

(a) Find its height at t = 2 sec

(b) Find the velocity equation.

(c) Find the velocity with which the ball was

kicked ( ie the initial velocity at t = 0)

(d) Find the value of t when the ball is at its

highest point.

(e) Find the maximum height reached.

(f) Find the two times that the ball is at a

height of 35 metres.

(h) Use the quadratic formula to find the two

times that the height of the ball is 50 m.

4. A boomerang is thrown at high speed and

in the hands of an expert it returns to the

thrower and stops.

The flight of such a throw can be described

by the equation:

L = t( t – 9 )2  where L is the horizontal

distance in metres at t seconds

(a) Sketch a graph for values of t from 0 to 9.

L

t

(b) Find an expression for the velocity, v, at

t sec. and draw the velocity-time graph.

(c) Find the value of t when the boomerang is

at its furthest distance from the thrower.

(d) Find the maximum distance the

boomerang goes from the thrower.

(e) What is the speed at which the

boomerang is thrown ?

(f) Find its velocity at t = 8.5 sec

(ie half a sec before it stops.)

(g) Find the acceleration equation of the

boomerang and draw the acceleration-

time graph.

(h) At what time is the acceleration zero?

**ANSWERS**

1. A drop of petrol drops on the surface of

a pond and spreads in a circular shape.

The radius, r mm, at t sec is given by :

r = 5t + 4

(a) What was the initial radius of the drop

of petrol at the instant it hit the water?

(ie find r at t = 0) r = 4 mm

(b) Find the radius at t = 3 sec. r = 19 mm

(c) Find the **rate** at which the radius is

increasing.( ie dr = 5 mm/sec

dt

(d) Find an expression for the

circumference C of the petrol in terms

of t. (ie substitute r = 5t + 4 in the

equation C = 2 π r )

C = 2 π (5t + 4)

C = 10π t + 8π

(e) Find the rate of increase of the

circumference.( ie dC = 10π mm2/sec

dt

(f) Find an expression for the area A

of the petrol in terms of t.

(ie substitute r = 5t + 4 in the

equation A = π r2  = π (5t + 4)2

=π (25t2 + 40t + 16)

(g) Find the rate of increase of the area at

t sec. ( ie dA = π (50t + 40)

dt

(h) Find the rate of increase of the area at

t = 0 sec dA = π (50t + 40) = 40π

dt

(i) Find the rate of increase of the area at

t = 3 sec dA = π (50t+40) = 190π mm2/s

dt

2. A rocket is being launched and for the

first 10 seconds of its flight, its distance,

H, from the ground at t sec, is given by:

H = 2t3  metres

(a) Find H at t = 1 sec H = 2 m

(b) Find H at t = 10 sec H = 2000 m

(c) Find the Velocity equation for the

rocket at t sec. ( ie dH = V = 6t2

dt

(d) Find the velocity of the rocket at t = 1

V = 6 m/s

(e) Find the velocity of the rocket at t = 10

V = 600 m/s

(f) Find the acceleration equation for the

rocket. ( ie dV = a = 12t

dt

(g) Find the acceleration at t = 1 sec

a = 12 m/s/s

(h) Find the acceleration at t = 10 sec

a = 120 m/s/s

3. A ball is kicked vertically upwards and its

height H at t sec is given by H = 40t – 5t2

(a) Find its height at t = 2 sec

H = 80 – 20 = 60 m

(b) Find the velocity equation.

V = 40 – 10t

(c) Find the velocity with which the ball was

kicked ( ie the initial velocity at t = 0)

V = 40 m/s

(d) Find the value of t when the ball is at its

highest point.

When V = 0 40 – 10t = 0

t = 4 sec

(e) Find the maximum height reached.

At t = 4 H = 160 – 80 = 80 m

(f) Find the two times that the ball is at a

height of 35 metres.

35 = 40t – 5t2

5t2 – 40t + 35 = 0

5(t2 – 8t + 7) = 0

5(t – 1)(t – 7) = 0

t = 1 and 7 sec

(h) Use the quadratic formula to find the two

times that the height of the ball is 50 m.

50 = 40t – 5t2

5t2 – 40t + 50 = 0

t = 40 √(402 - 4× 5× 50)

10

t = 6.4 sec and 1.6 sec

4. A boomerang is thrown at high speed and

in the hands of an expert it returns to the

thrower and stops.

The flight of such a throw can be described

by the equation:

L = t( t – 9 )2  where L is the horizontal

distance in metres at t seconds

(a) Sketch a graph for values of t from 0 to 9.

L

9 t

(b) Find an expression for the velocity, v, at

t sec.

L = t( t2 – 18t + 81)

L = t3 – 18t2 + 81t

V = dL = 3t2 – 36t + 81

dt

= 3( t2 – 12t + 27)

= 3(t – 3 )( t – 9)

V

81

3 9 t

(c) Find the value of t when the boomerang is

at its furthest distance from the thrower.

When V = 0 t = 3 sec

(d) Find the maximum distance the

boomerang goes from the thrower.

Max L = 3 ( 3 – 9)2 = 108 m

(e) What is the speed at which the

boomerang is thrown ?

At t = 0 V = 81 m/s

(f) Find its velocity at t = 8.5 sec

(ie half a sec before it stops.)

V = 3(t – 3 )( t – 9)

= 3 ( 8.5 – 3 )( 8.5 – 9)

= – 8.25 m/s

(g) Find the acceleration equation of the

boomerang.

a = dV = 6t - 36

dt

a

3 6 9 t

-36

(h) At what time is the acceleration zero?

a = 0 if t = 6 sec