**YEAR 12: Rate of Change, Distance, Velocity and Acceleration.**

1. A drop of petrol drops on the surface of

 a pond and spreads in a circular shape.

 The radius, r cm , at t sec is given by :

 r = 5t + 4

(a) What was the initial radius of the drop

 of petrol at the instant it hit the water?

 (ie find r at t = 0)

(b) Find the radius at t = 3 sec.

(c) Find the **rate** at which the radius is

 increasing.( ie dr )

 dt

(d) Find an expression for the

 circumference C of the petrol in terms

 of t. (ie substitute r = 5t + 4 in the

 equation C = 2 π r )

(e) Find the rate of increase of the

 circumference.( ie dC )

 dt

(f) Find an expression for the area A

 of the petrol in terms of t.

 (ie substitute r = 5t + 4 in the

 equation A = π r2 )

(g) Find the rate of increase of the area at

 t sec. ( ie dA )

 dt

(h) Find the rate of increase of the area at

 t = 0 sec

(i) Find the rate of increase of the area at

 t = 3 sec

2. A rocket is being launched and for the

 first 10 seconds of its flight, its distance,

 H, from the ground at t sec, is given by:

 H = 2t3  metres

(a) Find H at t = 1 sec

(b) Find H at t = 10 sec

(c) Find the Velocity equation for the

 rocket at t sec. ( ie dH = V )

 dt

(d) Find the velocity of the rocket at t = 1

(e) Find the velocity of the rocket at t = 10

(f) Find the acceleration equation for the

 rocket. ( ie dV = a )

 dt

(g) Find the acceleration at t = 1 sec

(h) Find the acceleration at t = 10 sec

3. A ball is kicked vertically upwards and its

 height H at t sec is given by H = 40t – 5t2

(a) Find its height at t = 2 sec

(b) Find the velocity equation.

(c) Find the velocity with which the ball was

 kicked ( ie the initial velocity at t = 0)

(d) Find the value of t when the ball is at its

 highest point.

(e) Find the maximum height reached.

(f) Find the two times that the ball is at a

 height of 35 metres.

(h) Use the quadratic formula to find the two

 times that the height of the ball is 50 m.

4. A boomerang is thrown at high speed and

 in the hands of an expert it returns to the

 thrower and stops.

 The flight of such a throw can be described

 by the equation:

 L = t( t – 9 )2  where L is the horizontal

 distance in metres at t seconds

(a) Sketch a graph for values of t from 0 to 9.

 L

 t

(b) Find an expression for the velocity, v, at

 t sec. and draw the velocity-time graph.

(c) Find the value of t when the boomerang is

 at its furthest distance from the thrower.

(d) Find the maximum distance the

 boomerang goes from the thrower.

(e) What is the speed at which the

 boomerang is thrown ?

(f) Find its velocity at t = 8.5 sec

 (ie half a sec before it stops.)

(g) Find the acceleration equation of the

 boomerang and draw the acceleration-

 time graph.

(h) At what time is the acceleration zero?

**ANSWERS**

1. A drop of petrol drops on the surface of

 a pond and spreads in a circular shape.

 The radius, r mm, at t sec is given by :

 r = 5t + 4

(a) What was the initial radius of the drop

 of petrol at the instant it hit the water?

 (ie find r at t = 0) r = 4 mm

(b) Find the radius at t = 3 sec. r = 19 mm

(c) Find the **rate** at which the radius is

 increasing.( ie dr = 5 mm/sec

 dt

(d) Find an expression for the

 circumference C of the petrol in terms

 of t. (ie substitute r = 5t + 4 in the

 equation C = 2 π r )

 C = 2 π (5t + 4)

 C = 10π t + 8π

(e) Find the rate of increase of the

 circumference.( ie dC = 10π mm2/sec

 dt

(f) Find an expression for the area A

 of the petrol in terms of t.

 (ie substitute r = 5t + 4 in the

 equation A = π r2  = π (5t + 4)2

 =π (25t2 + 40t + 16)

(g) Find the rate of increase of the area at

 t sec. ( ie dA = π (50t + 40)

 dt

(h) Find the rate of increase of the area at

 t = 0 sec dA = π (50t + 40) = 40π

 dt

(i) Find the rate of increase of the area at

 t = 3 sec dA = π (50t+40) = 190π mm2/s

 dt

2. A rocket is being launched and for the

 first 10 seconds of its flight, its distance,

 H, from the ground at t sec, is given by:

 H = 2t3  metres

(a) Find H at t = 1 sec H = 2 m

(b) Find H at t = 10 sec H = 2000 m

(c) Find the Velocity equation for the

 rocket at t sec. ( ie dH = V = 6t2

 dt

(d) Find the velocity of the rocket at t = 1

 V = 6 m/s

(e) Find the velocity of the rocket at t = 10

 V = 600 m/s

(f) Find the acceleration equation for the

 rocket. ( ie dV = a = 12t

 dt

(g) Find the acceleration at t = 1 sec

 a = 12 m/s/s

(h) Find the acceleration at t = 10 sec

 a = 120 m/s/s

3. A ball is kicked vertically upwards and its

 height H at t sec is given by H = 40t – 5t2

(a) Find its height at t = 2 sec

 H = 80 – 20 = 60 m

(b) Find the velocity equation.

 V = 40 – 10t

(c) Find the velocity with which the ball was

 kicked ( ie the initial velocity at t = 0)

 V = 40 m/s

(d) Find the value of t when the ball is at its

 highest point.

 When V = 0 40 – 10t = 0

 t = 4 sec

(e) Find the maximum height reached.

 At t = 4 H = 160 – 80 = 80 m

(f) Find the two times that the ball is at a

 height of 35 metres.

 35 = 40t – 5t2

 5t2 – 40t + 35 = 0

 5(t2 – 8t + 7) = 0

 5(t – 1)(t – 7) = 0

 t = 1 and 7 sec

(h) Use the quadratic formula to find the two

 times that the height of the ball is 50 m.

 50 = 40t – 5t2

 5t2 – 40t + 50 = 0

 t = 40 √(402 - 4× 5× 50)

 10

 t = 6.4 sec and 1.6 sec

4. A boomerang is thrown at high speed and

 in the hands of an expert it returns to the

 thrower and stops.

 The flight of such a throw can be described

 by the equation:

 L = t( t – 9 )2  where L is the horizontal

 distance in metres at t seconds

(a) Sketch a graph for values of t from 0 to 9.

 L

 9 t

(b) Find an expression for the velocity, v, at

 t sec.

 L = t( t2 – 18t + 81)

 L = t3 – 18t2 + 81t

 V = dL = 3t2 – 36t + 81

 dt

 = 3( t2 – 12t + 27)

 = 3(t – 3 )( t – 9)

 V

 81

 3 9 t

(c) Find the value of t when the boomerang is

 at its furthest distance from the thrower.

 When V = 0 t = 3 sec

(d) Find the maximum distance the

 boomerang goes from the thrower.

 Max L = 3 ( 3 – 9)2 = 108 m

(e) What is the speed at which the

 boomerang is thrown ?

 At t = 0 V = 81 m/s

(f) Find its velocity at t = 8.5 sec

 (ie half a sec before it stops.)

 V = 3(t – 3 )( t – 9)

 = 3 ( 8.5 – 3 )( 8.5 – 9)

 = – 8.25 m/s

(g) Find the acceleration equation of the

 boomerang.

 a = dV = 6t - 36

 dt

 a

 3 6 9 t

 -36

(h) At what time is the acceleration zero?

 a = 0 if t = 6 sec