**RATE OF CHANGE PROBLEMS.**

KEY IDEA: The “***rate of increase of B” is dB***

 ***dt***

1. If ***B = 5t – 6*** find the rate of increase of B

2. If ***P = t2 + 3t*** find the rate of increase of P when t = 4

3. If ***V = t3 – 6t*** find the rate at which ***V*** is changing when:

 (i***) t = 10***

 (ii) ***t = 1***

***NB If the rate of change of V is POSITIVE then V is INCREASING.***

 ***If the rate of change of V is NEGATIVE then V is DECREASING.***

4.(a) Suppose a drop of water falls on a paper towel and the circular wet patch

 increases in radius according to the equation ***r = 2t + 3***

 where ***t is in seconds and r is the radius in mm.***

Find:

(i) The rate of increase of the radius in mm/sec

(ii) The rate of increase of the Circumference

 ***C = 2π r***

 ***= 2 π ( 2t + 3 )***

 ***=***

Rate of Inc = ***dC =*** mm/sec

 ***dt***

(iii) The rate of increase of the AREA of the wet patch at t = 1 sec:

 ***A = πr2***

 ***= π ( 2t + 3)2***

 ***= π ( 4t2 + 12t + 9)***

 =

Rate of increase is:

 ***dA =***

 ***dt***

5. Suppose a model car moves so that its distance, s , from O , at t sec is:

 ***s = t2 + 3t + 2 metres***

The “rate” at which the distance increases is what we call the **“velocity”**

so we can say, velocity ***v = ds metres per second or m/s or ms – 1***

 ***dt***

The “rate” at which the velocity increases is what we call the **“acceleration”**

so we can say, acceleration ***a = dV m/s every sec or m/s/s or ms – 2***

 ***dt***

If ***s = t2 + 3t + 2*** find the values of ***s, v and a at t = 4 sec***

***(i) s =***

***(ii) v =***

***(iii) a =***

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6. A ball is kicked vertically up into the air so that its height, h metres,

 at t sec is : ***h = 30t – 5t2***

(a) Find the velocity equation : ***V = dh***

 ***dt***

(b) Find the acceleration equation : ***a = dV***

 ***dt***

(c) Find the time when ***V = 0***

(d) Find the greatest height reached by the ball.

(e) Find the two times when ***h = 0***

(f) Find the two times when ***h = 20 metres.***

 ***(solve the quadratic equation using graphics calculator)***

7. A stone is thrown vertically up from the top of a cliff which is 25 metres

 high. The height of the stone from the top of the cliff at t secs is:

 ***h = 20t – 5t2***

Find (a) the velocity equation

 (b) the acceleration equation

 25m

 (c) the **initial velocity** of the stone

 (“initial” means at t = 0)

 (d) the time when the velocity is zero

 (e) the maximum height the stone reaches

 (f) the time taken for the stone to reach the bottom of the cliff.

 (d) the speed that the stone hits the beach at the bottom of the cliff.

8. A model car moves in a straight line and its distance from the starting point is

 given by : ***x = t ( t – 6)2***The motion lasts from ***t = 0 to t = 6*** sec

The graph would be a simple **cubic** curve. If ***x = t ( t – 6)2***

 ***= t( t2 – 12t + 36)***

 ***= t3 – 12t2  + 36***t

The velocity will be a simple **parabola**  ***v = 3t2 – 24t + 36***

 ***= 3(t2 – 8t + 12)***

 ***= 3(t – 2)(t – 6)***

and the acceleration will be a **line** graph. accel, ***a = 6t – 24***

The velocity will be zero when ***3t2 – 24t + 36 = 0***

 ***ie 3( ) = 0***

 ***ie 3( )( ) = 0***

 ***so t =***

Max distance from O will be when ***t =*** ………………

***Max dist = …………….. metres***.

 6

 2

 2

6

 ***x***

 32

 ***V***

 36

t

 t

4

4

4

t

***a***

-24

P

This car start from O with an initial velocity of 36 m/s and is slowing down until the speed becomes momentarily zero at t = 2 sec, a maximum distance of 32 m from O. The car then goes backwards reaching a max **speed** of 12 m/s at t = 4 sec

(actually, a min Velocity of –12) It then slows down reaching O , stopping at t = 6.

9. A “remote controlled car” sets off from the base of a wall at W and accelerates up to a certain maximum speed. It then decelerates until it momentarily comes to test and then accelerates in reverse along the same path, finally crashing into the wall at its original starting point W.

The equation for S, the distance from the wall in metres, at t seconds is :

 ***S = t2(9 – t) for 0 ≤ t ≤ 9***

(a) Find the velocity equation. S

 (i) Find the **initial** velocity

 (ii) Find the time at which ***v = 0***

 (iii) Find the car’s maximum

 distance from the wall.

 t

(b) Find the acceleration equation.

 v

 (i) Find the ***initial*** acceleration.

 (ii) Find the time at which the

 acceleration is zero. t

 (iii) Find the car’s maximum velocity.

 (iv) How far from the wall is the car

 when it stops accelerating and starts

 decelerating. (ie when ***a = 0***)

 a

(c) At what time does the car hit the wall ?

 t

(d) At what speed does the car hit the wall ?

(e) Draw careful graphs of **distance**, **velocity** and

 **acceleration** of the car for ***t = 0 to 9*** sec.