**FINDING WHERE THE GRADIENT OF A CURVE IS ZERO.**

Maximum and minimum points max

occur when the gradient is zero.

We must practise doing this process.

*At this stage we will not worry about*

*proving whether they are max or min points.*

 min

Find the *x* values of the points where

the gradient is zero.

**EXAMPLE**

**1.** Curve y = x3 – 3x2 – 9x + 4

 ***Grad y ′= 3x2 – 6x – 9***

 ***= 3(x2 – 2x – 3)***

 ***= 3(x – 3)(x + 1)***

 ***So Grad = 0 if x = 3 or – 1***

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2. Curve y = x2 – 10x + 11

***Grad y ′ = 2x – 10 = 0 at max/min***

 ***2x = 10***

 ***x = 5***

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3. Curve y = x3 + 6x2 – 15x + 7

***Grad y ′ = 3x2 + 12x – 15***

 ***= 3(x2 + 4x – 5)***

 ***= 3(x – 1)(x + 5) = 0***

 ***at max/min***

 ***x = 1 or – 5***

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4. Curve y = (x – 3)(x – 11)

 ***= x2 – 14x + 33***

***Grad y ′ = 2x – 14 = 0 at max/min***

 ***2x = 14***

 ***x = 7***

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5. Curve y = 3x(12 – x)

 ***= 36x – 3x2***

***Grad y ′ = 36 – 6x = 0 at max/min***

 ***36 = 6x***

 ***x = 6***

6. Curve y = x3 – x2 – x + 6

***Grad y ′ = 3x2 – 2x – 1***

 ***= (3x + 1)(x – 1) = 0***

 ***At max/min***

 ***x = – ⅓ or 1***

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7. Curve y = x3 + 2x2 + x – 5

***Grad y ′ = 3x2 + 4x + 1***

 ***= (3x + 1)(x + 1) = 0***

 ***At max/min***

***x = – ⅓ or –1***

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8. Curve y = 4x3 – 24x2 + 36x + 3

 ***Grad y ′ = 12x2 – 48x + 36***

 ***= 12(x2 – 4x + 3)***

 ***= 12(x – 1)(x – 3) = 0***

 ***At max/min***

***x = 1 or 3***

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9. Curve y = x(x – 12)2

 ***= x(x – 12) (x – 12)***

 ***= x(x2 – 24x + 144)***

 ***= x3 – 24x2 + 144x***

***Grad y ′ = 3x2 – 48x + 144***

 ***= 3(x2 – 16x + 48)***

 ***= 3(x – 4)(x – 12) = 0***

 ***At max/min***

 ***x = 4 or 12***