**Year 12 mathematics: INTERSECTIONS OF LINES, PARABOLAS AND CIRCLES: ANSWERS.**

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| *1.* ***y = x²***  ***y = x + 6***  *x² = x + 6*  *x² – x – 6 = 0*  *(x – 3)(x + 2)=0*  *x = 3 and x = –2*  *when x = 3, y = 9*  *when x = –2, y = 4*  *intersect at (3, 9) & (–2, 4)*  *2.* ***y = x² − 3x − 4***  ***y = x – 7***  *x – 7 = x² − 3x − 4*  *x² − 4x + 3 = 0*  *(x – 3)(x – 1) = 0*  *x = 3 and x = 1*  *when x = 3, y = –4*  *when x = 1, y = –6*  *intersect at (3, –4) & (1, –6)*  *3.* ***y = x² − 4***  ***y = 2x – 5***  *x² − 4 = 2x – 5*  *x² − 2x + 1 = 0*  *(x – 1)² = 0*  *x = 1*  *when x =1, y = 2×1–5 = –3*  *tangent at (1, –3)*  *4.* ***y = x²***  ***y = x – 4***  *x² = x – 4*  *x² – x + 4 = 0*  *no solution, no intersection*  *5****. y = x² + 2x – 8***  ***y = 2x – 4***  *x² + 2x – 8 = 2x – 4*  *x² – 4 = 0*  *(x + 2)(x – 2) = 0*  *x = –-2 and x = 2*  *when x = –2, y = – 8*  *when x = 2, y = 0*  *intersect at (2, 0) & (–2, –8)*  *6.* ***y = x² + x – 2***  ***y = 2x + p***  *x² + x – 2 = 2x + p*  *x² – x – 2 – p = 0*  *At point of tangency,*  *discriminant ∆ = 0*  *(-1)² – 4 ×1 × (-2 – p) = 0*  *1 + 8 + 4p = 0*  *p = –2.25* | *7.* ***y = 1 ↔ xy = 1***  ***x***  ***y = 2 – x***  *x(2 – x) = 1*  *2x – x² = 1*  *x² − 2x + 1 = 0*  *(x – 1)² = 0*  *x = 1*  *when x = 1, y = 1*  *tangent at (1,1)*  *8.* ***y = 1 ↔ xy = 1***  ***x***  ***y = 2x – 1***  *x(2x – 1) = 1*  *2x² − x −1 = 0*  *x = 1 and x = -0.5*  *when x = 1, y = 1*  *when x = -0.5, y = -2*  *intersect (1,1) & (-0.5, -2)*  *9.* ***y = 6 ↔ xy = 6***  ***x***  ***y = 7 – x***  *x(7 – x) = 6*  *7x – x² = 6*  *x² − 7x + 6 = 0*  *(x – 6)(x – 1) = 0*  *x = 6 and x = 1*  *when x = 6, y = 1*  *when x = 1, y = 6*  *intersect at (1, 6) and (6, 1)*  *10****. y = -4 ↔ xy = -4***  ***x***  ***y = x – 5***  *x(x – 5) = -4*  *x² − 5x + 4 = 0*  *(x – 1)(x – 4) = 0*  *x = 1 and x = 4*  *when x = 1, y = -4*  *when x = 4, y = -1*  *intersect at (1, -4) & (4, -1)* | *11.* ***y = 4 ↔ xy = 4***  ***x***  ***y = b – x***  *x(b – x) = 4*  *bx – x² = 4*  *x² − bx + 4 = 0*  *At point of tangency,*  *discriminant ∆ = 0*  *(-b)² − 4 ×1 ×4 = 0*  *b² − 16 = 0*  *b = -4 and b = 4*  *tangents when b=-4 & b=4.*  *12.* ***y = 2 ↔ xy = 2***  ***x***  ***y = mx + 8***  *x(mx + 8) = 2*  *mx² + 8x = 2*  *mx² + 8x − 2 = 0*  *At point of tangency,*  *discriminant ∆ = 0*  *8² – 4 ×m × -2 = 0*  *64 + 8m = 0*  *m = -8*  *13.* ***x² + y² = 25***  ***y = x – 1***  *x² + (x – 1) ² = 25*  *x² + x² − 2x + 1 = 25*  *2x² − 2x − 24 = 0*  *x = 4 and x = -3*  *when x = 4, y = 3*  *when x = -3, y = -4*  *intersect at (4, 3) & (-3, -4)*  *14.* ***x² + y² = 25***  ***y = 2x – 2***  *x² + (2x – 2)² = 25*  *x² + 4x² − 8x + 4 = 25*  *5x² − 8x – 21 = 0*  *x = 3 and x = -1.4*  *when x = 3, y = 4*  *when x = -1.4, y = -4.8*  *intersect at (3, 4) and*  *(-1.4, -4.8)*  *15****. x² + y² = 5***  ***y = x + 1***  *x² + (x + 1)² = 5*  *x² + x² + 2x + 1 = 5*  *2x² +2x – 4 = 0*  *x = 1 and x = -2*  *when x = 1, y = 2*  *when x = -2, y = -1*  *intersect at (1, 2) & (-2, -1)* | *16.* ***x² + y² = 13***  ***y = x + 1***  *x² + (x + 1)² = 13*  *x² + x² + 2x + 1 = 13*  *2x² + 2x – 12 = 0*  *x = 2 and x = -3*  *when x = 2, y = 3*  *when x = -3, y = -2*  *intersect at (2, 3) & (-3, -2)*  *17.* ***x² + y² = 10***  ***y = 3x***  *x² + (3x)² = 10*  *x² + 9x² = 10*  *10x² = 10*  *x = 1 and x = -1*  *when x = 1, y = 3*  *when x = -1, y = -3*  *intersect at (1, 3) & (-1, -3)*  *18.* ***x² + y² = 8***  ***y = x + 4***  *x² + (x + 4)² = 8*  *x² + x² + 8x + 16 = 8*  *2x² +8x + 8 = 0*  *x = -2, y = 2*  *tangent at (-2, 2)*  *19.* ***x² + y² = 2***  ***y = x + p***  *x² + (x + p)² = 2*  *x² + x² + 2px + p² − 2 = 0*  *2x² + 2px + p² − 2 = 0*  *At point of tangency,*  *discriminant ∆ = 0*  *(2p)² – 4× 2×(p² − 2) = 0*  *4p² − 8p² +16 = 0*  *-4p² = -16*  *p = 2 and p = -2*  *20****. x² + y² = 2***  ***y = 1***  ***x***  *x² + (1)² = 2*  *x*  *x4 + 1 = 2x²*  *x4 – 2x² + 1 = 0*  *x² = 1*  *x = 1 and x = -1*  *when x = 1, y = 1*  *when x = -1, y = -1*  *Intersect at (1,1) & (-1, -1)* |