YEAR 12. Maximum and Minimum PROBLEMS (ADVANCED STUDENTS)

1. An eight metre length of aluminium is used to make a window frame in the following design:

(a) Write an equation connecting x and b .
(b) Show that the area of the window frame can be written as :

$$
A=x(4-2 x)
$$

(c) Use Calculus to show that the area is a maximum when $\mathrm{x}=1 \mathrm{~m}$
(d) Find the maximum area of the frame.

## 2. A rectangular enclosure

 is to be made from 100 m of electric fence with 4 compartments to keep 4 animals apart as shown:

Using calculus, show that the max area possible is $250 \mathrm{~m}^{2}$
3. Another enclosure is to be erected using 30 m of electric fence using a wall as one of the sides. It has 2 compartments as shown:
 the max area possible is $75 \mathrm{~m}^{2}$
4. The outer case of a matchbox is made as shown in this cross section:


The total length when opened out is $3 \mathrm{x}+2 \mathrm{~b}=12 \mathrm{~cm}$.
To fit as many matches in as possible using the same amount of cardboard, the cross sectional area should be as large as possible. Prove that the max. cross sectional area is $6 \mathrm{~cm}^{2}$.
5. A 6 cm square plate of silver is to be made into a tray with as large a volume as possible. Small squares are cut off each corner and the sides are bent up to form a tray.

(a) Show that the volume is $V=4 x^{3}-24 x^{2}+36 x$
(b) Show that

$$
\frac{\mathrm{dV}}{\mathrm{dx}}=12\left(\mathrm{x}^{2}-4 \mathrm{x}+3\right)
$$

(c) Show that the maximum volume is when $\mathrm{x}=1 \mathrm{~cm}$.
(d) Find the max volume.
(e) What happens if $\mathrm{x}=3$ ?
(f) Sketch the graph of $\mathrm{V}=\mathrm{x}(6-2 \mathrm{x})^{2}=4 \mathrm{x}(\mathrm{x}-3)^{2}$ for x values from 0 to 3 showing the $\max / \mathrm{min}$ values of V .
6. A square sheet of thin steel 60 cm by 60 cm is to be folded into a closed box:

(a) Show that the volume can be written as :
$V=2 x^{3}-120 x^{2}+1800 x$
(b) Show that the maximum volume is when $\mathrm{x}=10 \mathrm{~cm}$
(c) Find the max volume.
7. When a factory produces $x$ articles per day, the profit $P$ is . $P=x^{3}-75 x^{2}+1800 x$
(The factory can produce any number up to 30 per day) Find how many should be produced to maximise the profit.
8.The efficiency E of a machine varies with the time $t$ it has been going.
The efficiency $\mathrm{E} \%$ at t hours is:

$$
\mathrm{E}=50-2 \mathrm{t}^{2}+20 \mathrm{t}
$$

(a) What is E at $\mathrm{t}=1$ ?
(b) At what time is $\mathrm{E}=92 \%$
(c) At what time is it working at maximum efficiency?
(d) Find the maximum efficiency level.
9. The temperature T in an experiment rises quickly then falls slowly to $0^{\circ} \mathrm{C}$ at $\mathrm{x}=12 \mathrm{~min}$ The formula for T at $\mathrm{x} \min$ is: $\mathrm{T}=\mathrm{x}(\mathrm{x}-12)^{2}$
(a) Show that $T^{\prime}=3(x-4)(x-12)$
(b) Find the max temperature.
(c) Draw a graph of the temperature change.

ANSWERS MAX/MIN Probs
1.
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b
(a) Write an equation connecting x and b .
$4 \mathrm{x}+2 \mathrm{~b}=8$

$$
b=4-2 x
$$

(b) Area $\mathrm{A}=\mathrm{x} \mathrm{b}$

$$
A=x(4-2 x)
$$

(c) $\mathrm{A}=4 \mathrm{x}-2 \mathrm{x}^{2}$
$A^{\prime}=4-4 x=0$ for $\max$
So $\mathrm{x}=1$
(d) maximum area $=2 \mathrm{~m}^{2}$
2. 100 m of electric fence b

$5 \mathrm{x}+2 \mathrm{~b}=100$ so $\mathrm{b}=50-2.5 \mathrm{x}$
$\mathrm{A}=\mathrm{x}(50-2.5 \mathrm{x})$
$=50 \mathrm{x}-2.5 \mathrm{x}^{2}$
$A^{\prime}=50-5 x=0$ for max
So $\mathrm{x}=10$ $\max$ area possible is $250 \mathrm{~m}^{2}$
3.

$\mathrm{A}=\mathrm{x}(30-3 \mathrm{x})$
$=30 x-3 x^{2}$
$\mathrm{A}^{\prime}=30-6 \mathrm{x}=0$ for max $\mathrm{x}=5$
max area is $5 \times 15=75 \mathrm{~m}^{2}$ 4.

$3 \mathrm{x}+2 \mathrm{~b}=12 \mathrm{~cm}$.

$$
\mathrm{b}=6-1.5 \mathrm{x}
$$

$A=x(6-1.5 x)$
$=6 x-1.5 x^{2}$
$\mathrm{A}^{\prime}=6-3 \mathrm{x}=0$ for $\max$
$\mathrm{x}=2$
So max. cross sectional area is $2 \times 3=6 \mathrm{~cm}^{2}$.

(a) $V=x(6-2 x)(6-2 x)$

$$
=x\left(36-24 x+4 x^{2}\right)
$$

(b) $V^{\prime}=12 x^{2}-48 x+36$
$V^{\prime}=12\left(x^{2}-4 x+3\right)$
(c) maximum volume is
when $12\left(x^{2}-4 x+3\right)=0$
$12(x-1)(x-3)=0$
$x=1$ for $\max (x=3$ for $\min )$
(d) $\max$ volume $=16 \mathrm{~cm}^{3}$
6. A square sheet of thin steel 60 cm by 60 cm is to be folded into a closed box:
(a) $V=x(30-x)(60-2 x)$

$$
\begin{aligned}
& =\mathrm{x}\left(1800-120 \mathrm{x}+2 \mathrm{x}^{2}\right. \\
& =2 \mathrm{x}^{3}-120 \mathrm{x}^{2}+1800 \mathrm{x}
\end{aligned}
$$

(b) $\mathrm{V}^{\prime}=6 \mathrm{x}^{2}-240 \mathrm{x}+1800$ maximum volume is when
$6 x^{2}-240 x+1800=0$
$6\left(x^{2}-40 x+300\right)=0$
$6(x-10)(x-30)=0$
$\mathrm{x}=10$ for max or 30 for min
(c) Find the max volume.
$V \max =10(20)(40)=8000 \mathrm{~cm}^{3}$
7. $P=x^{3}-75 x^{2}+1800 x$
$\mathrm{P}^{\prime}=3 \mathrm{x}^{2}-150 \mathrm{x}+1800=0 \max$
$3\left(x^{2}-50 x+600\right)=0$
$3(x-20)(x-30)=0$

$$
V=4 x^{3}-24 x^{2}+36 x
$$

$\mathrm{x}=20$ for $\max 30$ for min
8. $\mathrm{E}=50-2 \mathrm{t}^{2}+20 \mathrm{t}$
(a) $\mathrm{at} \mathrm{t}=1 \quad \mathrm{E}=68 \%$
(b) $\mathrm{E}=92=50-2 \mathrm{t}^{2}+20 \mathrm{t}$
$2 \mathrm{t}^{2}-20 \mathrm{t}+42=0$
$2\left(\mathrm{t}^{2}-10 \mathrm{t}+21\right)=0$
$2(t-7)(t-3)=0$
$\mathrm{t}=3, \mathrm{t}=7$
(c) maximum efficiency when $\mathrm{E}^{\prime}=0=20-4 \mathrm{t}$ so $\mathrm{t}=5$
(d) maximum efficiency $=100 \%$
$9 \mathrm{~T}=\mathrm{x}(\mathrm{x}-12)^{2}$
(e) if $x=3$ there is no silver left to make a tray!
(f) Sketch the graph of $V=x(6-2 x)^{2}=4 x(x-3)^{2}$

(a) $T=x\left(x^{2}-24 x+144\right)$
$=x^{3}-24 x^{2}+144 x$
$\mathrm{T}^{\prime}=3 \mathrm{x}^{2}-48 \mathrm{x}+144$
$\mathrm{T}^{\prime}=3(\mathrm{x}-4)(\mathrm{x}-12)$
(b) max temperature when $t=4$

Tmax $=256^{\circ} \mathrm{C}$


