## YEAR 12. Maximum and Minimum PROBLEMS

1. An eight metre length of aluminium is used to make a window frame in the following design:

(a) Write an equation connecting x and b .
(b) Show that the area of the window frame can be written as : $\mathrm{A}=\mathrm{x}(4-2 \mathrm{x})$
(c) Use Calculus to show that the area is a maximum when $\mathrm{x}=1 \mathrm{~m}$
(d) Find the maximum area of the frame.
2. A rectangular enclosure is to be made from 100 m of electric fence with 4 compartments to keep 4 animals apart as shown:


Using calculus, show that the max area possible is $250 \mathrm{~m}^{2}$
4. A 6 cm square plate of silver is to be made into a tray with as large a volume as possible. Small squares are cut off each corner and the sides are bent up to form a tray.

(a) Show that the volume is

$$
V=4 x^{3}-24 x^{2}+36 x
$$

(b) Show that $V^{\prime}=12\left(x^{2}-4 x+3\right)$
(c) Show that the maximum volume is when $x=1 \mathrm{~cm}$.
(d) Find the max volume.
(e) What happens if $\mathrm{x}=3$ ?

ANSWERS MAX/MIN Probs

b
(a) Write an equation connecting x and b .
$4 \mathrm{x}+2 \mathrm{~b}=8$
$b=4-2 x$
(b) Area $A=x$ b

$$
A=x(4-2 x)
$$

(c) $A=4 x-2 x^{2}$
$A^{\prime}=4-4 x=0$ for max So $\mathrm{x}=1$
(d) maximum area $=2 \mathrm{~m}^{2}$
2. 100 m of electric fence b

$5 \mathrm{x}+2 \mathrm{~b}=100$
so $b=50-2.5 \mathrm{x}$
$\mathrm{A}=\mathrm{x}(50-2.5 \mathrm{x})$

$$
=50 \mathrm{x}-2.5 \mathrm{x}^{2}
$$

$A^{\prime}=50-5 \mathrm{x}=0$ for max
So $\mathrm{x}=10$
max area possible is $250 \mathrm{~m}^{2}$


$$
\begin{aligned}
A & =x(30-3 x) \\
& =30 x-3 x^{2}
\end{aligned}
$$

$\mathrm{A}^{\prime}=30-6 \mathrm{x}=0$ for max

$$
x=5
$$

max area is $5 \times 15=75 \mathrm{~m}^{2}$

(a) $V=x(6-2 x)(6-2 x)$
$=x\left(36-24 x+4 x^{2}\right)$
$V=4 x^{3}-24 x^{2}+36 x$
(b) $V^{\prime}=12 x^{2}-48 x+36$
$V^{\prime}=12\left(x^{2}-4 x+3\right)$
(c) maximum volume is when $12\left(x^{2}-4 x+3\right)=0$
$12(x-1)(x-3)=0$
$\mathrm{x}=1$ for $\max (\mathrm{x}=3$ for $\min )$
(d) $\max$ volume $=16 \mathrm{~cm}^{3}$
(e) if $x=3$ there is no silver left to make a tray!

