## A FARMER'S MAXIMUM AREA PROBLEM.

A farmer wants to make 3 enclosures to separate her calves into males and females. The females will be kept as future milk producers and the males will be going to the freezing works!

The farmer has 120 metres of a special temporary fencing material. She wants to make 3 areas as shown below:

Section	Section	Section
F	A	M

All the calves are firstly put in Section A then the females are sorted into Section F and the males to Section M.

The farmer wants the area to be as big as possible.

At first she thinks the dimensions should be as follows:



We can see that the sum of all the pieces of fencing is 120 metres. The area enclosed is  $10 \times 40 = 400 \text{ m}^2$ .

Her mathematician friend tells her, he can find a bigger area using calculus. Show clearly how this would be done.



$$b + 4x = 120$$
  

$$b + 2x = 60$$
  

$$b = 60 - 2x$$

Area  $A = xb = x(60 - 2x) = 60x - 2x^2$  A' = 60 - 4x = 0 for maximum area x = 15 metres and b = 60 - 30 = 30 metres this configuration produces the greatest area of  $A = 15 \times 30 = 450$  m<sup>2</sup> The farmer decides she wants an even bigger area by using an existing fence in the paddock as one of the sides as follows:





Use calculus to work out the new maximum area for the 3 enclosures.

b=120-4x

Area 
$$A = xb = x(120 - 4x)$$
  
 $A = 120x - 4x^2$   
 $A' = 120 - 8x = 0$  for maximum area  
 $x = \frac{120}{8} = 15$  metres and  $b = 60$  metres  
New max area =  $15 \times 60 = 900$  m<sup>2</sup>