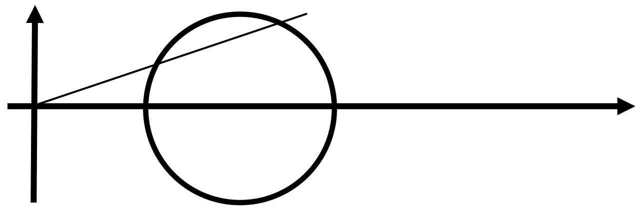
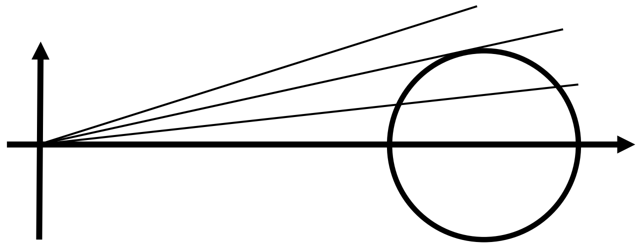
THREE INVESTIGATIONS.

1(a) Find the coordinates of the points where the line $y = \frac{1}{2}x$ crosses the circle given by $(x - 4)^2 + y^2 = 8$

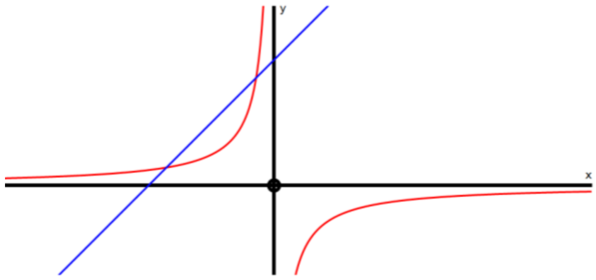


(b)



A line of the form $y = mx$ can cross the circle $(x - 4)^2 + y^2 = 8$ once or twice or not at all.
Find the value of m so that the line is a tangent.

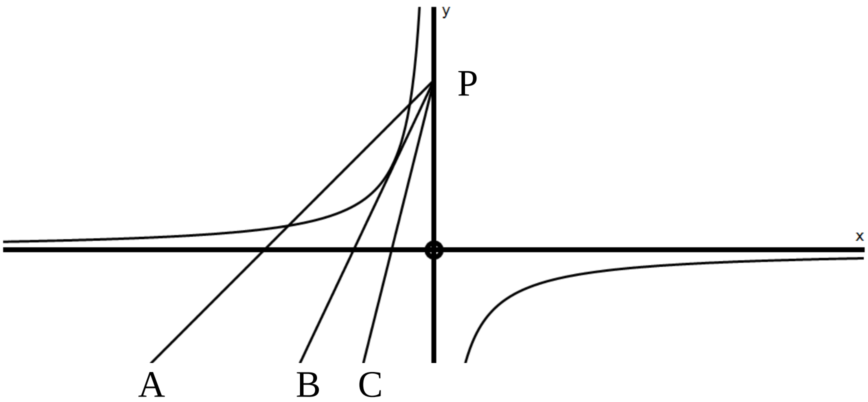
2(a)



The graphs above show the intersection points of $y = -\dfrac{6}{x}$ and $y = x + 5$

Find the coordinates of the intersection points algebraically.

(b) Consider the line $y = mx + 5$ which has a fixed y intercept at P but a variable gradient m.

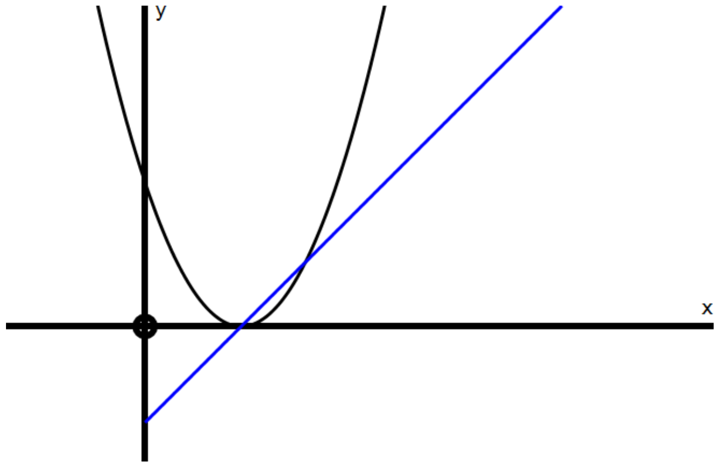


Line A crosses the hyperbola twice, Line B is a tangent and Line C does not cross.
Find the value of m so that $y = mx + 5$ is a tangent.

(c) Write down the range of values for m so that there will be 2 intersections.

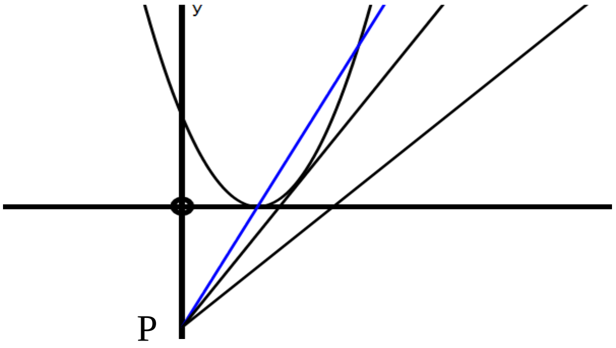
(d) Write down the range of values for m so that there will be no intersections.

3(a) The graphs below show $y = (x - 3)^2$ *and* $y = x - 3$



Calculate algebraically the intersection points.

(b) Consider the line $y = mx - 7$ which has a fixed y intercept but a variable gradient m.

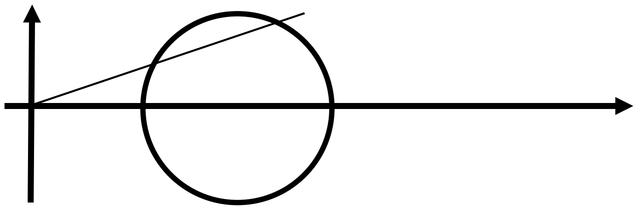


Find the value of m so that $y = mx - 7$ is a tangent to the parabola $y = (x - 3)^2$

(c) If $y = mx - k$ is to be a tangent to $y = (x - k)^2$ show that $m^2 + 4km - 4k = 0$

THREE INVESTIGATIONS.ANSWERS

1(a) Find the coordinates of the points where the line $\boldsymbol{y = \frac{1}{2}x}$ crosses the circle given by $\boldsymbol{(x - 4)^2 + y^2 = 8}$



Subs $\boldsymbol{y = \frac{1}{2}x}$ into $\boldsymbol{(x - 4)^2 + y^2 = 8}$ ***and we get*** $\boldsymbol{(x - 4)^2 + \frac{x^2}{4} = 8}$

$$\boldsymbol{x^2 - 8x + 16 + \frac{x^2}{4} = 8}$$

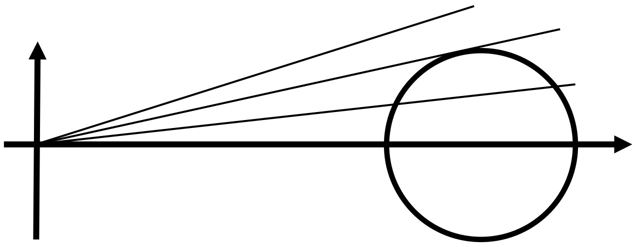
$$\boldsymbol{4x^2 - 32x + 64 + x^2 = 32}$$

$$\boldsymbol{5x^2 - 32x + 32 = 0}$$

$$\boldsymbol{x = 1.24 \text{ and } 5.16}$$

intersection points are (1.24, 0.62) and (5.16, 2.58)

(b)



A line of the form $\boldsymbol{y = mx}$ can cross the circle $\boldsymbol{(x - 4)^2 + y^2 = 8}$ once or twice or not at all.

Find the value of $\boldsymbol{m}$ so that the line is a tangent.

Subs $\boldsymbol{y = mx}$ into $\boldsymbol{(x - 4)^2 + y^2 = 8}$ ***and we get*** $\boldsymbol{(x - 4)^2 + m^2x^2 = 8}$

$$\boldsymbol{x^2 - 8x + 16 + m^2x^2 = 8}$$

$$\boldsymbol{(1 + m^2)\, x^2 - 8x + 8 = 0}$$

This equation will only have 1 solution so the discriminant = 0

$$\boldsymbol{\Delta = 64 - 4\times(1 + m^2)\times 8 = 0}$$

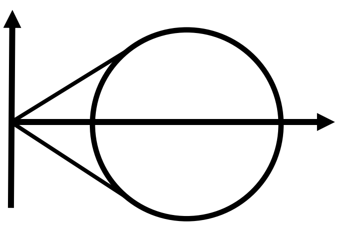
$$\boldsymbol{64 - 32 - 32m^2 = 0}$$

$$\boldsymbol{32 = 32\, m^2}$$

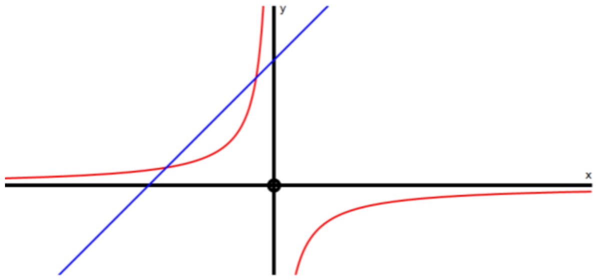
$$\boldsymbol{m^2 = 1}$$

so m could be 1 or – 1

There are 2 possible tangents y = x and y = –x



2(a)



The graphs above show the intersection points of $y = -\frac{6}{x}$ and $y = x + 5$

Find the coordinates of the intersection points algebraically.

$$x + 5 = -\frac{6}{x}$$

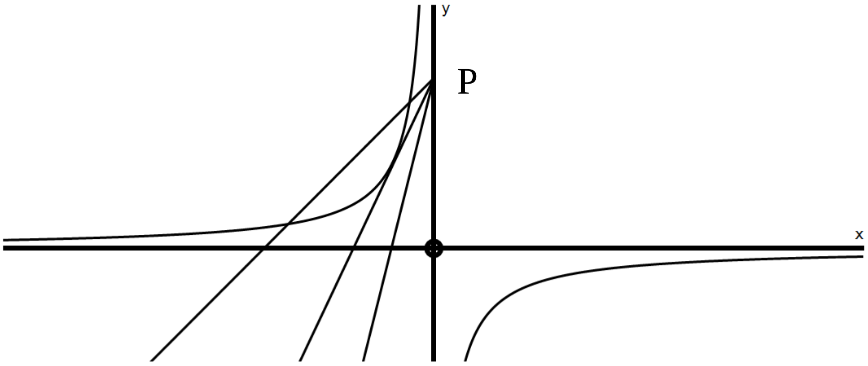
$$x^2 + 5x = -6$$

$$x^2 + 5x + 6 = 0$$

$$(x + 2)(x + 3) = 0$$

$x = -2$ and -3 The intersection points are $(-2, 3)$ and $(-3, 2)$

(b) Consider the line $y = mx + 5$ which has a fixed y intercept at P but a variable gradient m.



Line A crosses the hyperbola twice, Line B is a tangent and Line C does not cross.

Find the value of m so that $y = mx + 5$ is a tangent.

$$mx + 5 = -\frac{6}{x}$$

$$mx^2 + 5x + 6 = 0$$

to be a tangent $\Delta = 0$ *so* $25 - 24m = 0$ *giving* $m = \frac{25}{24}$

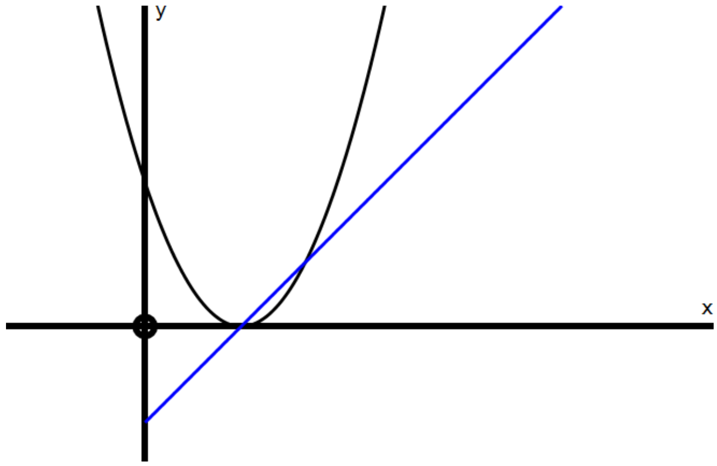
(c) Write down the range of values for m so that there will be 2 intersections.

$\Delta > 0$ *so* $25 - 24m > 0$ *giving* $25 > 24m$ *so* $\frac{25}{24} > m$

(d) Write down the range of values for m so that there will be no intersections.

$\Delta < 0$ *so* $25 - 24m < 0$ *giving* $25 < 24m$ *so* $\frac{25}{24} < m$

3(a) The graphs below show $y = (x - 3)^2$ *and* $y = x - 3$



Calculate algebraically the intersection points.

$x^2 - 6x + 9 = x - 3$

$x^2 - 7x + 12 = 0$

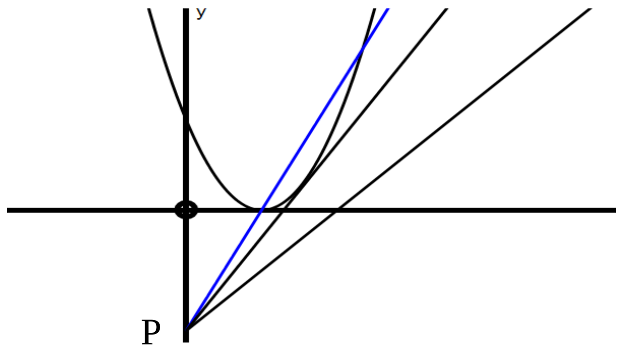
$(x - 3)(x - 4)=0$

So $x = 3$ ***and*** $x = 4$

The intersection points are:

$(3, 0)$ ***and*** $(4, 1)$

(b) Consider the line $y = mx - 7$ which has a fixed y intercept but a variable gradient m.



Find the value of m so that $y = mx - 7$ is a tangent to the parabola $y = (x - 3)^2$

$x^2 - 6x + 9 = mx - 7$

$x^2 - (m + 6)x + 16 = 0$

to be a tangent) = 0

so $(m + 6)^2 - 64 = 0$

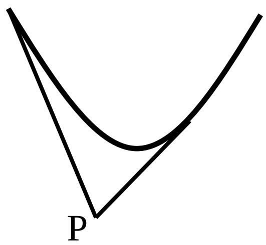
$(m + 6) = 64$

$m + 6 = \pm 8$

hence $m = 2$ ***or*** $m = -14$

This means there are 2 tangents from P above.

The tangents are $y = 2x - 7$ ***and*** $y = -14x - 7$



(c) If $y = mx - k$ is to be a tangent to $y = (x - k)^2$ show that $m^2 + 4km - 4k = 0$

$x^2 - 2kx + k^2 = mx - k$

$x^2 - (2k + m)x + (k^2 + k) = 0$

For the line to be a tangent, this equation must have 1 solution so its discriminant = 0

$\Delta = (2k + m)^2 - 4(k^2 + k) = 0$

$4k^2 + 4km + m^2 - 4k^2 - 4k = 0$

So $m^2 + 4km - 4k = 0$