Solutions (or Roots) of Quadratics.

If the roots of a quadratic are α and β then the equation must be :
$(x - \alpha)(x - \beta) = 0$

Multiplying out, we get:
$x^2 - \alpha x - \beta x + \alpha\beta = 0$
$x^2 - (\alpha + \beta) x + \alpha\beta = 0$
so for ***any quadratic equation in the form $x^2 + bx + c = 0$***
the PRODUCT of the roots $\alpha\beta = c$
and the SUM of the roots $\alpha + \beta = -b$

eg1. for the equation $x^2 + 9x + 20 = 0$
$\alpha\beta = 20$
$\alpha + \beta = -9$

eg2. for this equation $x^2 - 5x - 6 = 0$
$\alpha\beta = -6$
$\alpha + \beta = +5$

eg3. for the equation:$3x^2 - 5x + 11 = 0$
first we should divide by 3
$x^2 - \frac{5}{3}x + \frac{11}{3} = 0$
so $\alpha\beta = \frac{11}{3}$
$\alpha + \beta = \frac{5}{3}$

1. Suppose we say that the roots of the equation $x^2 - 3x + 2 = 0$ are α and β then FIND the equation with roots of 2α and 2β.

Firstly $\boldsymbol{\alpha\beta = 2}$
and $\boldsymbol{\alpha + \beta = 3}$

The equation would be :
$(x - 2\alpha)(x - 2\beta) = 0$
Multiplying out, we get:
$x^2 - 2\alpha x - 2\beta x + 4\alpha\beta = 0$
$x^2 - 2(\alpha + \beta) x + 4\alpha\beta = 0$
subs $\alpha\beta = 2$
and $\alpha + \beta = 3$
we get :
$x^2 - 2(3)x + 4(2) = 0$
$x^2 - 6x + 8 = 0$

Or we just say:
Product = $2\alpha\times2\beta$
$=4\times\alpha\beta = \mathbf{8}$
Sum $=2\alpha+2\beta$
$=2(\alpha+\beta)= \mathbf{6}$

2. Suppose we say that the roots of the equation $x^2 - 3x + 2 = 0$ are α and β then FIND the equation with roots of ***3α*** *and* ***3β.***

Firstly $\alpha\beta = 2$
and $\alpha + \beta = 3$
The equation would be :
$(x - 3\alpha)(x - 3\beta) = 0$
Multiplying out, we get:
$x^2 - 3\alpha x - 3\beta x + 9\alpha\beta = 0$
$x^2 - 3(\alpha + \beta) x + 9\alpha\beta = 0$
subs $\alpha\beta = 2$
and $\alpha + \beta = 3$
we get :
$x^2 - 3(3)x + 9(2) = 0$
$x^2 - 9x + 18 = 0$

Or we just say:
Product = $3\alpha\times3\beta$
$=9\times\alpha\beta = \mathbf{18}$
Sum $=3\alpha+3\beta$
$=3(\alpha+\beta)= \mathbf{9}$

3. Suppose we say that the roots of the equation ***$x^2 + bx + c = 0$*** *are α and β then FIND the equation with roots of* ***4α*** *and* ***4β.***

Firstly $\alpha\beta = c$
and $\alpha + \beta = -b$
The equation would be :
$(x - 4\alpha)(x - 4\beta) = 0$
Multiplying out, we get:
$x^2 - 4\alpha x - 4\beta x + 16\alpha\beta = 0$
$x^2 - 4(\alpha + \beta) x + 16\alpha\beta = 0$
subs $\alpha\beta = c$
and $\alpha + \beta = -b$
we get :
$x^2 - 4(-b)x + 16(c) = 0$
$\boldsymbol{x^2 + 4bx + 16c = 0}$

Or we just say:
Product = $4\alpha\times4\beta$
$=16\times\alpha\beta$
$= \mathbf{16c}$
Sum $=4\alpha+4\beta$
$=4(\alpha+\beta)$
$= \mathbf{-4b}$