## Y12: EXCELLENCE LEVEL B

## ALGEBRA.

1. Find the value of the constant " $c$ " so that the line $\boldsymbol{y}=\mathbf{3 x}+\boldsymbol{c}$ is a tangent to the

$$
\begin{array}{cc}
\text { curve } y=-\frac{12}{x} & \text { If } 3 x+c=-12 / x \\
3 x^{2}+c x=-12 \\
3 x^{2}+c x+12=0 \\
\text { If tangent then discrim })=0 \\
c^{2}-4.3 .12=0 \\
c^{2}=144 \\
c= \pm 12
\end{array}
$$

2. A Biological researcher found that the number of bacteria in a culture could be calculated at some future time using a formula of the form $\boldsymbol{N}=\boldsymbol{A} \times \boldsymbol{b}^{t}$
Where $\boldsymbol{N}=$ the number of bacteria at $\boldsymbol{t}$ hours. $\boldsymbol{A}$ and $\boldsymbol{b}$ are unknown constants.
She estimated that at $t=4$ hours, $N$ was 5,600 and at $t=7$ hours, $N$ was $\mathbf{5 9 , 7 0 0}$.
Use this information to calculate the constants $\boldsymbol{A}$ and $\boldsymbol{b}$ then use your formula to estimate the number of bacteria at $t=12$ hours.
Subs $5600=A . b^{4} \quad$ and $\quad 59700=A . b^{7}$
$\begin{aligned} & 59700 \\ & 5600=\frac{A \cdot b^{7}}{A \cdot b^{4}} \quad \text { so } b^{3}=\frac{59700}{5600} \quad \text { so } \quad b=2.2 \quad \text { sub } 5600=A 2.2^{4} \\ & A=239\end{aligned}$
$N=239 \times 2.2^{12}=3072345$

## CALCULUS

2. A 40 cm piece of wire is cut into two pieces.

The first piece is shaped into a circle of area A , and the second piece into a square of area B.
Find the minimum value of the total area $\mathrm{A}+\mathrm{B}$.
Lengths are $x$ and $40-x$ if circumf $=x$ then $2 \pi r=x$ so $r=x /(2 \pi)$ Area $A+B=\pi r^{2}+\frac{(40-x)^{2}}{16}$

$$
\begin{aligned}
& \text { AREA }=\frac{\pi x^{2}}{4 \pi^{2}}+\frac{1600-80 x+x^{2}}{16}=\frac{x^{2}}{4 \pi}+100-5 x+\frac{x^{2}}{16} \\
& \frac{d(\text { Area })}{d x}=\frac{x}{2 \pi}-5+\frac{x}{8}=0 \text { for max Area } \\
& x\left(\frac{1}{2 \pi}+\frac{1}{8}\right)=5
\end{aligned}
$$

$$
x \times 0.28415=5
$$

$$
x=17.6 \mathrm{~cm} \text { so } r=2.8
$$

So Min Area $=\pi \times 2.8^{2}+5.6^{2}=56 \mathrm{~cm}^{2}$

