PARALLEL ALGEBRA ASSESSMENT ANSWERS

1.(a) *Simplify fully:*

$$\frac{(x^2 - x - 12)}{(x^2 - 16)} = \frac{(x-4)(x+3)}{(x-4)(x+4)} = \frac{(x+3)}{(x+4)}$$

(b) *Expand and simplify:* $(2x - 5)(x + 2)(x - 3)$

$$= (2x - 5)(x^2 - x - 6)$$

$$= 2x^3 - 2x^2 - 12x - 5x^2 + 5x + 30$$

$$= 2x^3 - 7x^2 - 7x + 30$$

(c) *Factorise fully:*

(i) $3x^2 - 3x - 6 = 3(x^2 - x - 2) = 3(x - 2)(x + 1)$

(ii) $3x^2 - 11x - 4 = (3x + 1)(x - 4)$

(d) (i) *Change this equation into the form* $ax^2 + bx + c = 0$

$(x - 2)(x + 1) = 2x + 5$

$x^2 - x - 2 - 2x - 5 = 0$

$x^2 - 3x - 7 = 0$

(ii) *Solve the equation in (i) using the quadratic formula and give your answer to 3 sig fig.*

$a = 1 \quad b = -3 \quad c = -7$

$$x = \frac{3 \pm \sqrt{(9 + 4.1.7)}}{2} = \frac{3 \pm \sqrt{37}}{2} = 4.54 \text{ or } -1.54$$

(e) *The equation* $2x^2 + 3x - (k + 2) = 0$ ***has only one real solution. Find k.***

Has 1 sol if $\Delta = 0$ so $9 + 4 \times 2 \times (k + 2) = 0$

$9 + 8k + 16 = 0$

$8k = -25$

$k = -\frac{25}{8}$

(f) (i) Express $px^2 - 4x + p = x^2 - 1$ in the form $ax^2 + bx + c = 0$

$$px^2 - x^2 - 4x + p + 1 = 0$$

$$(p - 1)x^2 - 4x + (p + 1) = 0$$

(ii) Find the possible values of p so that the equation in (i) has real solutions.

Has real solutions (ie 2 real or 1 real) if $\Delta \geq 0$

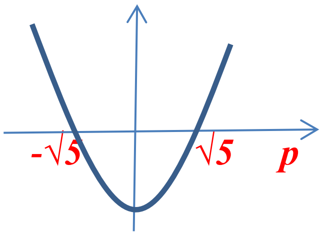
$$16 - 4(p - 1)(p + 1) \geq 0$$

$$16 - 4(p^2 - 1) \geq 0$$

$$16 - 4p^2 + 4 \geq 0$$

$$20 \geq 4p^2$$

$$5 \geq p^2$$



$$-\sqrt{5} \leq x \leq \sqrt{5}$$

2.(a) Solve for x: $4(3x - 2) = 6 - x$

$$12x - 8 = 6 - x$$

$$13x = 14$$

$$x = \frac{14}{13}$$

(b) Solve for x: $(x - 1)^2 - 2(x + 2) + 7 = 0$

$$x^2 - 2x + 1 - 2x - 4 + 7 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)^2 = 0$$

$$x = 2$$

(c) Solve for x: $3x^2 = x + 2$

$$3x^2 - x - 2 = 0$$

$$(3x + 2)(x - 1) = 0$$

$x = -2/3$ or 1 or on Graph Calc $x = 1$ or -0.667

(d) The area of the outside of a cone is given by $A = \pi RL$ where R is the radius of the base and L is the slant height. If $L = R - 2$ and the area is 24π, find the values of R and L.

$$\pi RL = 24\pi$$

$$\pi R(R - 2) = 24\pi$$

$$R(R - 2) = 24$$

$$R^2 - 2R - 24 = 0$$

$$(R - 6)(R + 4) = 0$$

$R = 6$ but $R = -4$ is not valid

Hence $R = 6$, $L = 4$

(e) Solve the equation: $\dfrac{1}{x} + \dfrac{1}{(x+3)} = \dfrac{13}{40}$

Mult both sides by 40x(x + 3)

$40(x+3) + 40x = 13x(x+3)$

$40x + 120 + 40x = 13x^2 + 39x$

$0 = 13x^2 - 41x - 120$

$0 = (13x + 24)(x - 5)$

$x = -24/13$ or 5

on graphic calc $x = 5$ or -1.85

3. (a) Factorise $7x^2 - 4x + 3 = (7x + 3)(x - 1)$

(b) Solve $7x^2 - 4x + 3 = 0$

$(7x + 3)(x - 1) = 0$

$x = 1$ or $-7/3$

(c) Find the EXACT solution of the equation:
$x^2 - 8x = 5$ in the form $d \pm \sqrt{p}$ using the quadratic formula or by completing the square method

$x^2 - 8x + 16 = 5 + 16$ **OR** $x^2 - 8x - 5 = 0$

$(x-4)^2 = 21$ $\qquad x = \dfrac{8 \pm \sqrt{(64 + 4\times1\times5)}}{2}$

$x - 4 = \pm\sqrt{21}$

$x = 4 \pm \sqrt{21}$ $\qquad x = \dfrac{8 \pm \sqrt{(84)}}{2}$

(These answers are equal)

(d) Find what the value of k must be: $\dfrac{x^2 - 5x + k}{x^2 - 6x + 8} = \dfrac{x-3}{x-4}$

$\dfrac{x^2 - 5x + k}{(x-2)(x-4)} = \dfrac{x-3}{x-4}$

Clearly (x – 2) must cancel from the left hand side and leave (x – 3) on top.
so (x – 2) must be a factor of $x^2 - 5x + k$ and other factor is (x – 3)
so $(x-2)(x-3) = x^2 - 5x + 6$ *ie* $k = 6$

(e) Bob tries to solve the equation: $\dfrac{x^2 - 2x - 24}{x^2 - 6x} = 3$

This is his working:

$x^2 - 2x - 24 = 3x^2 - 18x$	line 1
$0 = 2x^2 - 16x + 24$	line 2
$0 = 2(x^2 - 8x + 12)$	line 3
$0 = 2(x - 3)(x - 4)$	line 4
$x = -3$ or -4	line 5

(i) Find which lines Bob has made mistakes. LINES 4 and 5

(ii) Explain what the mistakes are.

He factorised wrongly $2(x^2 - 8x + 12) = 2(x - 2)(x - 6)$

and the solutions from this are $x = 2$ or 6

BUT $x = 6$ is not valid because of the denominator $x^2 - 6x = 0$

and we cant have 0 on the bottom line of a fraction.

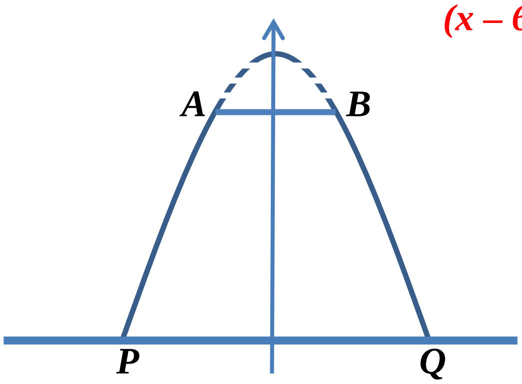
(iii) Suggest another way he could have solved the equation.

Factorising firstly: $\dfrac{(x + 4)(x - 6)}{x(x - 6)} = 3$

$$x + 4 = 3x$$
$$4 = 2x$$
$$2 = x$$

But $x \neq 6$ because we cancelled $\dfrac{(x - 6)}{(x - 6)}$

(f)



This building has a parabolic cross section with the top chopped off.
The distance AB is 20 metres.
The distance PQ is 40 metres.
The height of AB above the ground PQ is 60 metres.

Find the equation of this parabolic cross section and find how high it would have been if the top were not chopped off.

Equation is of the form $y = c - bx^2$

If $x = 20$, $y = 0$ so $0 = c - 400b$

If $x = 10$, $y = 60$ so $60 = c - 100b$

Subtracting $60 = 300b$ so $b = 1/5$

Subs $0 = c - \dfrac{400}{5} = 80$

The equation is $y = 80 - \dfrac{x^2}{5}$

Full height would have been 80 metres.