

Useful teaching examples for finding Max/Min values.

$$1. \ y = x^2 - 6x + 8$$

$$y' = 2x - 6 = 0 \text{ for min}$$

$$x = 3$$

$$2. \ y = x^2 + 10x + 9$$

$$y' = 2x + 10 = 0 \text{ for min}$$

$$x = -5$$

$$3. \ y = x^2 - 5x + 11$$

$$y' = 2x - 5 = 0 \text{ for min}$$

$$x = 2.5$$

$$4. \ y = x^3 - 3x^2 - 45x + 26$$

$$y' = 3x^2 - 6x - 45 = 0$$

$$3(x^2 - 2x - 15) = 0$$

$$3(x - 5)(x + 3) = 0$$

$$x = 5(\text{min}) \ x = -3(\text{max})$$

$$5. \ y = x^3 - 6x^2 - 36x + 58$$

$$y' = 3x^2 - 12x - 36 = 0$$

$$3(x^2 - 4x - 12) = 0$$

$$3(x - 6)(x + 2) = 0$$

$$x = 6(\text{min}) \ x = -2(\text{max})$$

$$6. \ y = x^3 - x^2 - 21x + 78$$

$$y' = 3x^2 - 2x - 21 = 0$$

$$(3x + 7)(x - 3) = 0$$

$$x = 3(\text{min}) \ x = -\frac{7}{3}(\text{max})$$

$$7. \ y = x^3 + 2x^2 - 7x + 98$$

$$y' = 3x^2 + 4x - 7 = 0$$

$$(3x + 7)(x - 1) = 0$$

$$x = -\frac{7}{3}(\text{max}) \ x = 1(\text{min})$$

$$8. \ y = 4x^3 - 24x^2 - 144x + 4$$

$$y' = 12x^2 - 48x - 144 = 0 \text{ at max/min}$$

$$12(x^2 - 4x - 12) = 0$$

$$12(x - 6)(x + 2) = 0$$

$$x = 6(\text{min}), \ x = -2(\text{max})$$

$$9. \ y = -x(x - 6)^2$$

$$= -x(x^2 - 12x + 36)$$

$$= -x^3 + 12x^2 - 36x$$

$$y' = -3x^2 + 24x - 36$$

$$= -3(x^2 - 8x + 12)$$

$$= -3(x - 2)(x - 6) = 0 \text{ at TP}$$

$$x = 2(\text{min}), \ x = 6(\text{max})$$

$$10. \ y = x(x + 3)^2$$

$$= x(x^2 + 6x + 9)$$

$$= x^3 + 6x^2 + 9x$$

$$y' = 3x^2 + 12x + 9 = 0$$

$$3(x^2 + 4x + 3) = 0$$

$$3(x + 1)(x + 3) = 0$$

$$x = -1(\text{min}) \ x = -3(\text{max})$$

$$11. \ y = x^2(x - 6)$$

$$= x^3 - 6x^2$$

$$y' = 3x^2 - 12x = 0 \text{ at max/min}$$

$$3x(x - 4) = 0$$

$$x = 0(\text{max}) \ x = 4(\text{min})$$

$$12. \ y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x + 9$$

$$y' = x^2 + x - 12 = 0 \text{ at max/min}$$

$$(x - 3)(x + 4) = 0$$

$$x = 3(\text{min}), \ x = -4(\text{max})$$

$$13. \ y = 4x^2 - \frac{2}{3}x^3$$

$$y' = 8x - 2x^2 = 0 \text{ at max/min}$$

$$= 2x(4 - x) = 0$$

$$x = 0(\text{min}) \ x = 4(\text{max})$$