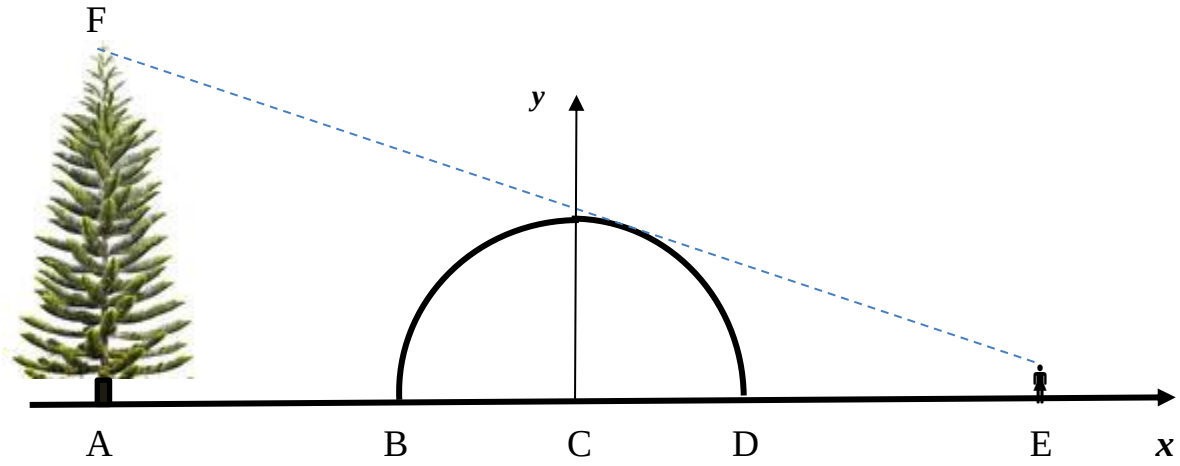
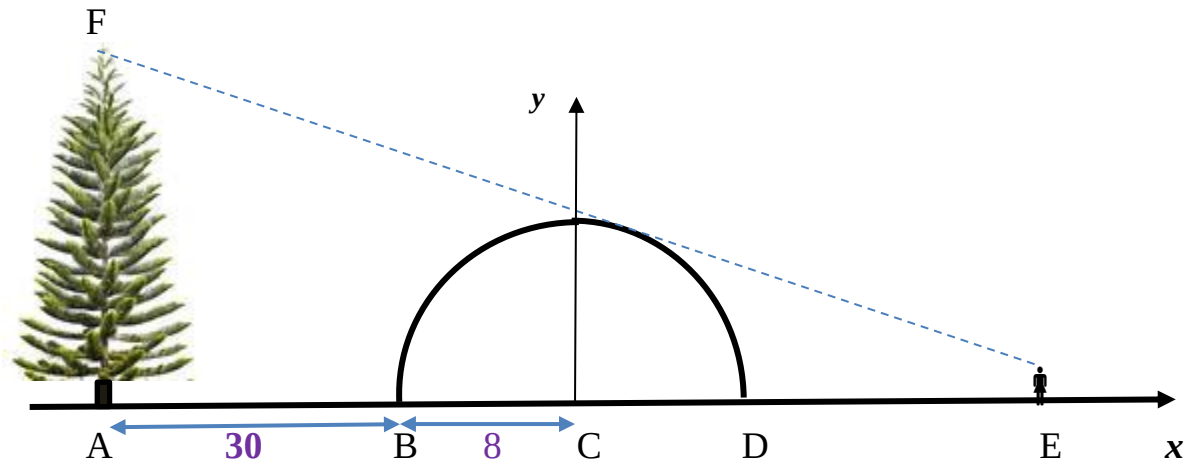
The Norfolk Pine Tree.



The large Norfolk pine tree is 30 metres from the side of the hemispherical dome-house. AB = 30 m
The diameter of the dome-house is 16 metres.
The tree has grown so much in the last year that it is now just visible from the point E which is on the other side of the dome-house.
The distance DE is **not known** but the gradient of the "line of sight" from E to the top of the tree F, is $-\frac{1}{2}$.
Calculate the height of the Norfolk pine tree.

The Norfolk Pine Tree. ANSWERS



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The line FE will be a tangent to the dome-house.
The equation will be $y = -\frac{1}{2}x + c$
The equation of the dome-house is $x^2 + y^2 = 64$
Solving simultaneously: $x^2 + (-\frac{1}{2}x + c)^2 = 64$

$$x^2 + \frac{x^2}{4} - cx + c^2 = 64$$

$$4x^2 + x^2 - 4cx + 4c^2 = 256$$

$$5x^2 - 4cx + (4c^2 - 256) = 0$$

For the line of sight to be a tangent $\Delta = 0$ ***so there is only 1 solution.***

$$16c^2 - 4 \times 5 \times (4c^2 - 256) = 0$$

$$16c^2 - 80c^2 + 5120 = 0$$

$$5120 = 64c^2$$

$$c^2 = 80$$

$$c = 8.94$$

(the negative answer refers to the tangent to the whole circle underground!)
The equation of FE is $y = -\frac{1}{2}x + 8.94$

The value of x at A is $x = -38$ ***so substituting:***
The height of the tree is $y = -\frac{1}{2} \times -38 + 8.94$

$$= 27.94 \text{ metres}$$