## SUMMARY OF CALCULUS 2.

1. Find the gradient of $y=5+7 x+x^{4}$ when $x=1$
$y^{\prime}=7+4 x^{3}$
sub $x=1$ and $y^{\prime}=11$
2. State whether the graph of $y=x^{3}-12 x^{2}+36 x$ is increasing, decreasing or stationary
$y^{\prime}=3 x^{2}-24 x+36=3\left(x^{2}-8 x+12\right)$

$$
=3(x-2)(x-6)
$$

when
(a) $x=4$
$y^{\prime}=$ neg so decreasing
(b) $x=2$
$y^{\prime}=0$ so stationary
(c) $x=7$
$y^{\prime}=$ positive so increasing
3. Find the $x$ coordinates of the turning
points (ie max/ min points) of the graph
$y=x^{2}(x-4)^{2}$
$y=x^{2}\left(x^{2}-8 x+16\right)$
$y=x^{4}-8 x^{3}+16 x^{2}$
$y^{\prime}=4 x^{3}-24 x^{2}+32 x=0$ at max $/ \mathrm{min}$
$=4 x\left(x^{2}-6 x+8\right)$
$=4 x(x-2)(x-4)$
$x=0,2,4$
Using $2^{\text {nd }}$ derive test:
$y^{\prime \prime}=12 x^{2}-48 x+32$
if $x=0$ then $y^{\prime \prime}=$ positive so min point
if $x=2$ then $y^{\prime \prime}=$ negative so $\max p t$
if $x=4$ then $y^{\prime \prime}=$ positive so min point
4. Consider this piecewise graph:


Draw the gradient function:

5. If $y^{\prime}=-2 x+5$
find the equation for $y$ if the curve goes through $(1,7)$
$y=-x^{2}+5 x+c$
sub $x=1, y=7$
$7=-1+5+c$
$3=c$
Equ is $y=-x^{2}+5 x+3$
6. Find the equation of the tangent to $y=x^{2}-2 x+1$ at the point where $x=3$ so $y=4$
$y^{\prime}=2 x-2$ sub $x=3$ so $y^{\prime}=4$
tan is of form $y=m x+c$
so $4=4 \times 3+c$
$c=-8$
tan is $y=4 x-8$
7. The distance, $x$ of an object from $O$ at $t$ secs is given by :
$x=t^{2}-2 t+4$
(a) how far from $O$ is it at $t=0$ sec? $x=4$ metres
(b) how far from $O$ is it at $t=5 \mathrm{sec}$ ? $x=25-10+4=19$ metres
(c) find the velocity equation

$$
v=\frac{d x}{d t}=2 t-2
$$

(d) find the velocity at $t=0 \mathrm{sec}$ $v=-2 \mathrm{~m} / \mathrm{s}$
(e) find the velocity at $t=5 \mathrm{sec}$ $v=8 \mathrm{~m} / \mathrm{s}$
(f) find the acceleration equation

$$
a=\frac{d v}{d t}=2 \mathrm{~m} / \mathrm{s} / \mathrm{s}
$$

8. A gun is fired so that the bullet goes vertically upwards.
The height of the bullet at $t \mathrm{sec}$ is $H=80 t-5 t^{2}+2$
(a) how high was the gun as the bullet was fired?
When $t=0, \quad H=2$ metres
(b) find the velocity equation.
$v=80-10 t$
(c) find the time when the velocity of the bullet was zero.
$80-10 t=0$ so $t=8 \mathrm{sec}$
(d) find the greatest height reached by the bullet.
$H=80 \times 8-5 \times 8^{2}+2=322$ metres
(e) what was the initial velocity of the bullet?
$t=0$ so $v=80 \mathrm{~m} / \mathrm{sec}$
(f) how far did the bullet travel in the $4^{\text {th }}$ second? (from $t=3$ to $t=4$ ).
When $t=3, H=197 \mathrm{~m}$
When $t=4, H=242 \mathrm{~m}$
So dist $=242-197=45$ metres
9. The velocity of an object is given by $v=4 t^{3}-8 t \mathrm{~m} / \mathrm{s}$
Find the distance equation given that at $t=2$ sec the distance $x=10 \mathrm{~m}$ $x=t^{4}-4 t^{2}+c$
sub $t=2, x=10$
$10=16-16+c$
So $c=10$
So $x=t^{4}-4 t^{2}+10$
10. This is the gradient graph of a function $y=f(x)$


Draw the function:


