## Towards Excellence.

1. 


(b)(i) If $y=-x+c$ is to be a tangent to $y=\underline{8}$ find c
(ii) Explain fully why there are two values for $\mathbf{c}$.
(c) Find all the values of c for which the line $\boldsymbol{y}=\boldsymbol{x}+\boldsymbol{c}$ will cross the Hyperbola $y=\frac{8}{x}$ at exactly two points.
(d) Find all the values of $c$ for which the line $\boldsymbol{y}=\boldsymbol{x}+\boldsymbol{c}$ will NOT cross the Hyperbola $y=\frac{8}{x}$ at all.
2. The graph shown has the equation $\boldsymbol{y}=\underline{\boldsymbol{\sigma}} \boldsymbol{x}+\boldsymbol{\sigma}$

If a line $\boldsymbol{y}=m x$ is to be a tangent to $\boldsymbol{y}=\underline{\boldsymbol{\sigma}} \boldsymbol{x}+\boldsymbol{\sigma}$ find $\boldsymbol{m}$.

3. The graph below has the equation $y=\frac{-12}{x}+9$

If a tangent has a gradient of 2 , find the coordinates of the point where the tangent meets the $\boldsymbol{x}$ axis.


## Explain clearly why there are TWO answers!

Towards Excellence. ANSWERS
1.

(a) Find algebraically, the intersection points of the graphs $y=\underline{8}$ and $y=-x+6$

$$
\underline{8}=-x+6
$$

$$
\begin{aligned}
& x \\
& 8=-x^{2}+6 x
\end{aligned}
$$

$$
x^{2}-6 x+8=0
$$

$$
(x-2)(x-4)=0
$$

$$
x=2 \text { and } x=4
$$

$$
y=4 \text { and } y=2
$$

$$
\text { intersection points are }(2,4) \text { and }(4,2)
$$

(b)(i) If $y=-x+c$ is to be a tangent to $y=\underline{8}$ find c $x$
$\underline{8}=-x+c$
$\boldsymbol{x}$
$8=-x^{2}+c x$
$x^{2}-c x+8=0$
if there is to be 1 solution then the discriminant $=0$ so $c^{2}-4 \times 8=0$ $c^{2}=32$
$c \quad \approx \pm 5.66$

(ii) Explain fully why there are two values for c .

A line with a gradient of 2 can be a tangent to each half of the hyperbola. See diagram above.
(c) Find all the values of c for which the line $\boldsymbol{y}=\boldsymbol{x}+\boldsymbol{c}$ will cross the Hyperbola $y=\frac{8}{x}$ at exactly two points.
Line will cross twice if there are 2 solutions to $x^{2}-c x+8=0$ and this occurs if the discriminant is positive so $c^{2}-4 \times 8>0$

$$
c^{2}>32
$$

$$
\text { so } c<-5.66 \text { or } c>+5.66
$$

(d) Find all the values of $c$ for which the line $\boldsymbol{y}=\boldsymbol{x}+\boldsymbol{c}$ will NOT cross the Hyperbola $y=\underline{8}$ at all.

$$
x
$$

Line will not cross if there are no solutions to $x^{2}-c x+8=0$ and this occurs if the discriminant is negative so $c^{2}<32$

$$
\text { so }-5.66<c<+5.66
$$

2. The graph shown has the equation $\boldsymbol{y}=\underline{\boldsymbol{6}}+\boldsymbol{6}$

If a line $\boldsymbol{y}=\boldsymbol{m} \boldsymbol{x}$ is to be a tangent to $\boldsymbol{y}=\frac{\boldsymbol{\sigma}}{\boldsymbol{x}}+\boldsymbol{\sigma}$ find $\boldsymbol{m}$.


$$
\begin{aligned}
& m x=\frac{6}{x}+6 \\
& m x^{2}=6+6 x \\
& m x^{2}-6 x-6=0
\end{aligned}
$$

there will be only 1 solution if )=0
so $36-4 m(-6)=0$
$24 m=-36$
$m=\frac{-3}{2}$
3. The graph below has the equation $y=\underline{\mathbf{- 1 2}}+9$

If a tangent has a gradient of 2 , find the coordinates of the point where the tangent meets the $\boldsymbol{x}$ axis.


> Let tan be $y=2 x+c$
> Intersection is found by solving:
> $2 x+c=\frac{-12}{x}+9$
> $2 x^{2}+c x=-12+9 x$
> $2 x^{2}+x(c-9)+12=0$
> Only 1 sol if $)=0$
> $(c-9)^{2}-4 \times 2 \times 12=0$
> $(c-9)^{2}=96$
> $c=9 \pm 9.8$
> $c=-0.8$ or 18.8

Explain clearly why there are TWO answers!
There are two tangents because if the gradient of a line is 2 it can be a tangent to each half of the hyperbola as shown on the diagram.
The 2 equations are $y=2 x-0.8$ and $y=2 x+18.8$
These cross the $x$ axis when the $y$ value is 0 ie at $x=0.4$ and $x=-9.4$

