YEAR 12 QUADRATIC THEORY ANS

1(a) Solve by **factorising** :

$x^2 + 5x - 14 = 0$

(x – 2)(x + 7) = 0

x = 2 or – 7

(b) Solve by using the method called "**completing the square**" and show each step clearly.

$x^2 + 8x = 33$

$x^2 + 8x + 16 = 33 + 16$

$(x + 4)^2 = 49$

x + 4 = 7 or x + 4 = – 7

x = 3 or –11

2. Given $ax^2 + bx + c = 0$

then $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Use the quadratic formula or **graphics calculator** to solve these equations and give your solutions to 2 dec pl.

(a) $3x^2 + 9x + 5 = 0$

x = –0.74 or –2.26

(b) $5x^2 - 7x - 11 = 0$

x = 2.34 or –0.94

3. Show clearly how to solve each of the following 4 equations by completing the square (even though 2 of them factorise) and state how the discriminant affects the type of solutions.

(a) $x^2 - 8x + 7 = 0$

$x^2 - 8x = -7$

$x^2 - 8x + 16 = 16 - 7$

$(x - 4)^2 = 9$

x – 4 = 3 or x – 4 = –3

x = 7 or 1

Δ = 64 – 28 = 36 so 2 rational sols

(b) $x^2 - 8x + 16 = 0$

$x^2 - 8x + 16 = 16 - 16$

$(x - 4)^2 = 0$

x = 4

Δ = 64 – 64 = 0 so 1 rational sol.

(c) $x^2 - 8x + 5 = 0$

$x^2 - 8x + 16 = 16 - 5 = 11$

$x = 4 \pm \sqrt{11}$

Δ = 64 – 20 = 44 so 2 irrational sols.

(d) $x^2 - 8x + 16 = 16 - 20 = -4$

$(x - 4)^2 = -4$

Can't find √– 4

Δ = 64 – 80 = – 16 so no real sols.

4. The Discriminant is $\Delta = b^2 - 4ac$.

State what **type** of solutions you get if the discriminant is :

(a) 0 ***= 1 rat sol***

(b) 36 ***= 2 rat sol***

(c) – 9 ***= no real sols but 2 complex***

(d) 3 ***= 2 irrat sol***

(e) 1 ***= 2 rat sols.***

Use the discriminant in the following :

5. Find c so that $x^2 - 12x + c = 0$ has 1 rational solution.

Δ = 144 – 4c = 0

4c = 144

c = 36

6. Find the **range of values** of p so that $x^2 - 10x + p = 0$ has no real solutions.

Δ = 100 – 4p < 0

100 < 4p

25 < p

7. Find n so that $2x^2 + nx + 8 = 0$ has only one rational solution.

Δ = n^2 – 64 = 0

n = ±8

8. Find k so that $x^2 + kx + (k + 3) = 0$ has only one rational solution.

Δ = k^2 – 4(k + 3) = 0

k^2 – 4k – 12 = 0

(k – 6)(k + 2) = 0

k = 6 or – 2

9. Find p so that $x^2 + (p+2)x + (3p-2) = 0$ has only one rational solution.

$\Delta = (p+2)^2 - 4(3p-2) = 0$

$p^2 + 4p + 4 - 12p + 8 = 0$

$p^2 - 8p + 12 = 0$

$(p-2)(p-6) = 0$

$p = 2$ or 6

10. Find d if $x^2 + (d+3)x + 3d + 1 = 0$ has only one rational solution.

$\Delta = (d+3)^2 - 4(3d+1) = 0$

$d^2 + 6d + 9 - 12d - 4 = 0$

$d^2 - 6d + 5 = 0$

$(d-1)(d-5) = 0$

$d = 1$ or 5

11. Find the range of values of K so that $x^2 - 8x + K = 0$ has no real solutions.

$\Delta = 64 - 4K < 0$

$64 < 4k$

$16 < k$

12. Find the range of values of b so that $x^2 + bx + 9 = 0$ has no real solutions.

$\Delta = b^2 - 36 < 0$

$b^2 < 36$

$b < +6$ or $b > -6$

can be written as $-6 < b < 6$

13. Find the range of values of n so that $x^2 + (n+2)x + (n+5) = 0$ has 2 real solutions.

$\Delta = (n+2)^2 - 4(n+5) > 0$

$n^2 + 4n + 4 - 4n - 20 > 0$

$n^2 - 16 > 0$

$n^2 > 16$

so $n > 4$ or $n < -4$

14. Find the range of values of p so that $x^2 + (p-1)x + p + 2 = 0$ has no real solutions.

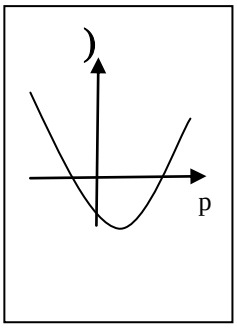
$\Delta = (p-1)^2 - 4(p+2) < 0$

$p^2 - 2p + 1 - 4p - 8 < 0$

$p^2 - 6p - 7 < 0$

$(p-7)(p+1) < 0$

So $-1 < p < 7$



15. Find k so that the equation $x^2 + 2(k-2)x + (k^2 - k - 5) = 0$ has only one rational solution.

$\Delta = 4(k-2)^2 - 4(k^2 - k - 5) = 0$

$4(k^2 - 4k + 4) - 4k^2 + 4k + 20 = 0$

$4k^2 - 16k + 16 - 4k^2 + 4k + 20 = 0$

$-12k + 36 = 0$

$k = 3$