

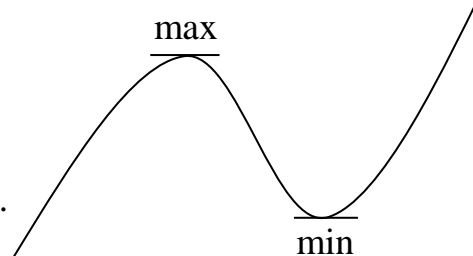
FINDING WHERE THE GRADIENT OF A CURVE IS ZERO.

Maximum and minimum points

occur when the gradient is zero.

We must practise doing this process.

At this stage we will not worry about proving whether they are max or min points.



Find the x values of the points where the gradient is zero.

EXAMPLE

1. Curve $y = x^3 - 3x^2 - 9x + 4$

$$\begin{aligned}\text{Grad } y' &= 3x^2 - 6x - 9 \\ &= 3(x^2 - 2x - 3) \\ &= 3(x - 3)(x + 1)\end{aligned}$$

So Grad = 0 if $x = 3$ or -1

2. Curve $y = x^2 - 10x + 11$

$$\begin{aligned}\text{Grad } y' &= 2x - 10 = 0 \text{ at max/min} \\ 2x &= 10 \\ x &= 5\end{aligned}$$

3. Curve $y = x^3 + 6x^2 - 15x + 7$

$$\begin{aligned}\text{Grad } y' &= 3x^2 + 12x - 15 \\ &= 3(x^2 + 4x - 5) \\ &= 3(x - 1)(x + 5) = 0 \\ &\text{at max/min} \\ x &= 1 \text{ or } -5\end{aligned}$$

4. Curve $y = (x - 3)(x - 11)$
 $= x^2 - 14x + 33$

$$\begin{aligned}\text{Grad } y' &= 2x - 14 = 0 \text{ at max/min} \\ 2x &= 14 \\ x &= 7\end{aligned}$$

5. Curve $y = 3x(12 - x)$
 $= 36x - 3x^2$

$$\begin{aligned}\text{Grad } y' &= 36 - 6x = 0 \text{ at max/min} \\ 36 &= 6x \\ x &= 6\end{aligned}$$

6. Curve $y = x^3 - x^2 - x + 6$

$$\begin{aligned}\text{Grad } y' &= 3x^2 - 2x - 1 \\ &= (3x + 1)(x - 1) = 0 \\ &\text{At max/min} \\ x &= -\frac{1}{3} \text{ or } 1\end{aligned}$$

7. Curve $y = x^3 + 2x^2 + x - 5$

$$\begin{aligned}\text{Grad } y' &= 3x^2 + 4x + 1 \\ &= (3x + 1)(x + 1) = 0 \\ &\text{At max/min} \\ x &= -\frac{1}{3} \text{ or } -1\end{aligned}$$

8. Curve $y = 4x^3 - 24x^2 + 36x + 3$

$$\begin{aligned}\text{Grad } y' &= 12x^2 - 48x + 36 \\ &= 12(x^2 - 4x + 3) \\ &= 12(x - 1)(x - 3) = 0 \\ &\text{At max/min} \\ x &= 1 \text{ or } 3\end{aligned}$$

9. Curve $y = x(x - 12)^2$

$$\begin{aligned}&= x(x - 12)(x - 12) \\ &= x(x^2 - 24x + 144) \\ &= x^3 - 24x^2 + 144x \\ \text{Grad } y' &= 3x^2 - 48x + 144 \\ &= 3(x^2 - 16x + 48) \\ &= 3(x - 4)(x - 12) = 0 \\ &\text{At max/min} \\ x &= 4 \text{ or } 12\end{aligned}$$