YEAR 12: Rate of Change, Distance, Velocity and Acceleration.

1. A drop of petrol drops on the surface of a pond and spreads in a circular shape. The radius, r cm , at t sec is given by :

$$r = 5t + 4$$

(a) What was the initial radius of the drop of petrol at the instant it hit the water? (ie find r at t = 0)

(b) Find the radius at t = 3 sec.

(c) Find the **rate** at which the radius is increasing.(ie $\frac{dr}{dt}$)

(d) Find an expression for the circumference C of the petrol in terms of t. (ie substitute r = 5t + 4 in the equation $C = 2\pi r$)

(e) Find the rate of increase of the circumference.(ie $\frac{dC}{dt}$)

(f) Find an expression for the area A of the petrol in terms of t. (ie substitute r = 5t + 4 in the equation $A = \pi r^2$)

(g) Find the rate of increase of the area at t sec. (ie $\frac{dA}{dt}$)

(h) Find the rate of increase of the area at t = 0 sec

(i) Find the rate of increase of the area at t = 3 sec

2. A rocket is being launched and for the first 10 seconds of its flight, its distance, H, from the ground at t sec, is given by:

$$H = 2t^3 \text{ metres}$$

(a) Find H at t = 1 sec

(b) Find H at t = 10 sec

(c) Find the Velocity equation for the rocket at t sec. (ie $\frac{dH}{dt} = V$)

(d) Find the velocity of the rocket at t = 1

(e) Find the velocity of the rocket at t = 10

(f) Find the acceleration equation for the rocket. (ie $\frac{dV}{dt} = a$)

(g) Find the acceleration at t = 1 sec

(h) Find the acceleration at t = 10 sec

3. A ball is kicked vertically upwards and its height H at t sec is given by $H = 40t - 5t^2$

(a) Find its height at $t = 2$ sec

(b) Find the velocity equation.

(c) Find the velocity with which the ball was kicked (ie the initial velocity at $t = 0$)

(d) Find the value of t when the ball is at its highest point.

(e) Find the maximum height reached.

(f) Find the two times that the ball is at a height of 35 metres.

(h) Use the quadratic formula to find the two times that the height of the ball is 50 m.

4. A boomerang is thrown at high speed and in the hands of an expert it returns to the thrower and stops.
The flight of such a throw can be described by the equation:
$L = t(t - 9)^2$ where L is the horizontal distance in metres at t seconds

(a) Sketch a graph for values of t from 0 to 9.

(b) Find an expression for the velocity, v, at t sec. and draw the velocity-time graph.

(c) Find the value of t when the boomerang is at its furthest distance from the thrower.

(d) Find the maximum distance the boomerang goes from the thrower.

(e) What is the speed at which the boomerang is thrown ?

(f) Find its velocity at $t = 8.5$ sec (ie half a sec before it stops.)

(g) Find the acceleration equation of the boomerang and draw the acceleration-time graph.

(h) At what time is the acceleration zero?

ANSWERS

1. A drop of petrol drops on the surface of a pond and spreads in a circular shape. The radius, r mm, at t sec is given by :

$$r = 5t + 4$$

(a) What was the initial radius of the drop of petrol at the instant it hit the water? (ie find r at t = 0) $r = 4$ mm

(b) Find the radius at t = 3 sec. $r = 19$ mm

(c) Find the **rate** at which the radius is increasing.(ie $\frac{dr}{dt} = 5$ mm/sec

(d) Find an expression for the circumference C of the petrol in terms of t. (ie substitute $r = 5t + 4$ in the equation $C = 2\pi r$)

$$C = 2\pi(5t + 4)$$
$$C = 10\pi t + 8\pi$$

(e) Find the rate of increase of the circumference.(ie $\frac{dC}{dt} = 10\pi$ mm^2/sec

(f) Find an expression for the area A of the petrol in terms of t. (ie substitute $r = 5t + 4$ in the equation $A = \pi r^2 = \pi(5t + 4)^2$

$$= \pi(25t^2 + 40t + 16)$$

(g) Find the rate of increase of the area at t sec. (ie $\frac{dA}{dt} = \pi(50t + 40)$

(h) Find the rate of increase of the area at t = 0 sec $\frac{dA}{dt} = \pi(50t + 40) = 40\pi$

(i) Find the rate of increase of the area at t = 3 sec $\frac{dA}{dt} = \pi(50t+40) = 190\pi$ mm^2/s

2. A rocket is being launched and for the first 10 seconds of its flight, its distance, H, from the ground at t sec, is given by:

$$H = 2t^3 \text{ metres}$$

(a) Find H at t = 1 sec $H = 2$ m

(b) Find H at t = 10 sec $H = 2000$ m

(c) Find the Velocity equation for the rocket at t sec. (ie $\frac{dH}{dt} = V = 6t^2$

(d) Find the velocity of the rocket at t = 1
$V = 6$ m/s

(e) Find the velocity of the rocket at t = 10
$V = 600$ m/s

(f) Find the acceleration equation for the rocket. (ie $\frac{dV}{dt} = a = 12t$

(g) Find the acceleration at t = 1 sec
$a = 12$ m/s/s

(h) Find the acceleration at t = 10 sec
$a = 120$ m/s/s

3. A ball is kicked vertically upwards and its height H at t sec is given by $H = 40t - 5t^2$

(a) Find its height at t = 2 sec
$H = 80 - 20 = 60$ m

(b) Find the velocity equation.
$V = 40 - 10t$

(c) Find the velocity with which the ball was kicked (ie the initial velocity at t = 0)
$V = 40$ m/s

(d) Find the value of t when the ball is at its highest point.
When $V = 0$ $\quad 40 - 10t = 0$
$t = 4$ sec

(e) Find the maximum height reached.
At t = 4 $H = 160 - 80 = 80$ m

(f) Find the two times that the ball is at a height of 35 metres.

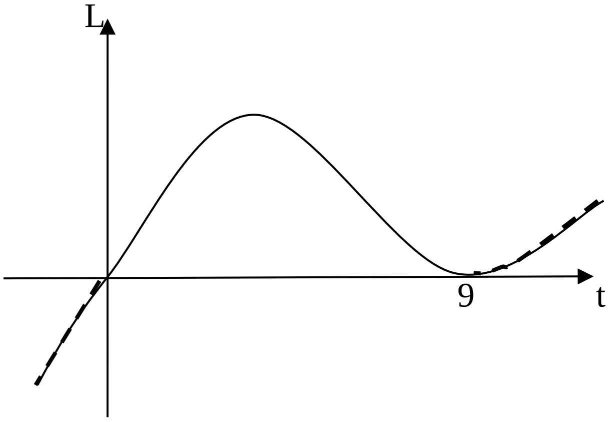
$$35 = 40t - 5t^2$$
$$5t^2 - 40t + 35 = 0$$
$$5(t^2 - 8t + 7) = 0$$
$$5(t - 1)(t - 7) = 0$$
$$t = 1 \text{ and } 7 \text{ sec}$$

(h) Use the quadratic formula to find the two times that the height of the ball is 50 m.

$$50 = 40t - 5t^2$$
$$5t^2 - 40t + 50 = 0$$
$$t = \frac{40 \pm \sqrt{(40^2 - 4\times 5\times 50)}}{10}$$
$$t = 6.4 \text{ sec and } 1.6 \text{ sec}$$

4. A boomerang is thrown at high speed and in the hands of an expert it returns to the thrower and stops.
The flight of such a throw can be described by the equation:
$L = t(t - 9)^2$ where L is the horizontal distance in metres at t seconds

(a) Sketch a graph for values of t from 0 to 9.



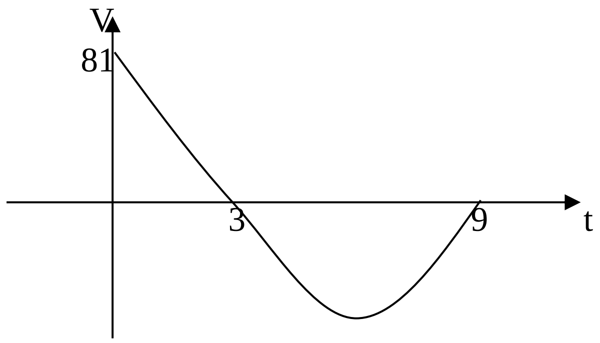
(b) Find an expression for the velocity, v, at t sec.

$L = t(t^2 - 18t + 81)$
$L = t^3 - 18t^2 + 81t$

$$V = \frac{dL}{dt} = 3t^2 - 36t + 81$$

$$= 3(t^2 - 12t + 27)$$

$$= 3(t - 3)(t - 9)$$



(c) Find the value of t when the boomerang is at its furthest distance from the thrower.

When $V = 0$ $t = 3$ sec

(d) Find the maximum distance the boomerang goes from the thrower.

Max $L = 3(3 - 9)^2 = 108$ m

(e) What is the speed at which the boomerang is thrown ?

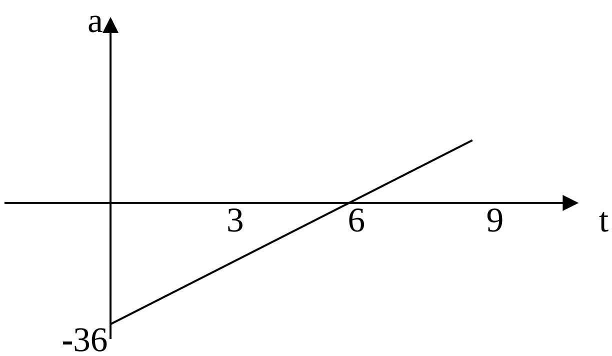
At $t = 0$ $V = 81$ m/s

(f) Find its velocity at $t = 8.5$ sec (ie half a sec before it stops.)

$V = 3(t - 3)(t - 9)$
$= 3(8.5 - 3)(8.5 - 9)$
$= -8.25$ m/s

(g) Find the acceleration equation of the boomerang.

$$a = \frac{dV}{dt} = 6t - 36$$



(h) At what time is the acceleration zero?

$a = 0$ if $t = 6$ sec