Y12 : PRACTICE ASSESSMENT A. MERIT LEVEL ONLY.

Algebra.

1. Solve: $(2x-5)^2 + x^2 = 25$

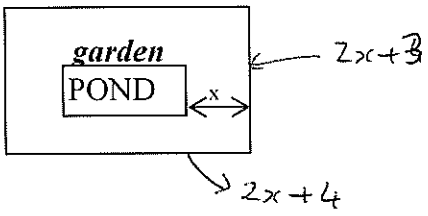
$4x^2 - 20x + 25 + x^2 = 25$

$5x^2 - 20x = 0$

$5x(x-4) = 0$

$x = 0, 4$

2.



The pond is 3m by 4m
The width of the garden is the same right round the pool.
The total area of the garden is 32 m^2
Find the width x of the garden to 3 sig fig.

$(2x+4)(2x+3) - 12 = 32$

$4x^2 + 14x + 12 - 12 = 32$

$4x^2 + 14x - 32 = 0$

$x = 1.58$ or -5.08

but x cannot be -5.08!

The only valid answer is $x = 1.58$ m

3. If I deposit $3000 for n years at 6% compound interest, find how many whole years it will take to more than double my money by solving:

$3000(1.06)^n = 6000$

$(1.06)^n = 2$

$\log(1.06)^n = \log 2$

$n \log 1.06 = \log 2$

$n = \frac{\log 2}{\log 1.06} = 11.896$

So it will take 12 years.

4. Solve $\frac{4x-8}{x+1} = x-2$

$4x - 8 = (x-2)(x+1)$

$4x - 8 = x^2 - x - 2$

$0 = x^2 - 5x + 6$

$0 = (x-2)(x-3)$

$x = 2, 3$

5. A man throws a cricket ball and the equation of its path is
$y = 2.3 + 5x - x^2/5$
where y is the height and x is the horizontal distance travelled in metres. Find how far from the man the ball lands.

Find when $y = 0$ of course.

$0 = 2.3 + 5x - \frac{x^2}{5}$

or $\frac{x^2}{5} - 5x - 2.3 = 0$

on calc.

$x = 40.46$ m

(Obviously, the other value -3.96 refers to the other part of the parabola before the ball was thrown!)

Calculus.

1. Find the turning points of the curve:
$y = 2x^3 - 9x^2 + 12x$
and determine their nature.

grad $y' = 6x^2 - 18x + 12 = 0$ at max/min

$6(x^2 - 3x + 2) = 0$

$(x-1)(x-2) = 0$

Cubics is this shape

So max is at $x = 1$, $y = 5$

min is at $x = 2$, $y = 4$

2. The height H metre of a metal ball shot into the air at t sec is given by:

$H = 80t - 5t^2$

(a) Find t when the ball is at its highest.

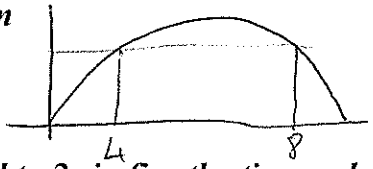
$\frac{dH}{dt} = 80 - 10t = 0$ at highest

So $t = 8$ sec.

(b) Find the greatest height the ball reaches

Sub $t = 8$, $H = 80 \times 8 - 5 \times 8^2 = 320$ m

(c) Find at what times the ball is at a height of 240m



$240 = 80t - 5t^2$

$5t^2 - 80t + 240 = 0$

$t = 4$ and 12 sec

(d) Find to 2 sig figs the times when the ball is at a height of 260 metres.

$260 = 80t - 5t^2$

$5t^2 - 80t + 260 = 0$

$t = 4.5$ sec and 11.5 sec.

3. If $y' = -3x^2 + 18x$ find y if $y = 4$ when $x = 2$

$y = -x^3 + 9x^2 + c$

Sub $4 = -8 + 36 + c$

$4 = 28 + c$

$-24 = c$

$y = -x^3 + 9x^2 - 24$

4. The velocity of a boomerang v at t sec is

$v = 30 - 6t$

(a) Find the initial velocity with which the boomerang was thrown.

Sub $t = 0$, $v = 30$ m/s

(b) At what time was it at its maximum distance away?

Max dist is when $v = 0$

$30 - 6t = 0$

So $t = 5$ sec.

(c) If x is the distance from the thrower find the maximum distance it goes.

$v = \frac{dx}{dt} = 30 - 6t$

$x = 30t - 3t^2 + c$

at $t = 0$, obviously $x = 0$

$x = 30t - 3t^2$

Sub $t = 5$

$x = 30 \times 5 - 3 \times 5^2$

$= 75$ m