YEAR 12 UNDERSTANDING LOGARITHMS - SOLUTIONS.

"Log" just means Index or power or exponent.

1. Consider $2^3 = 8$

the index $= 3$
ie the Log $= 3$
or in full $\text{Log}_2 8 = 3$

2. Write in log form as shown above.

(a) $4^2 = 16$ $\quad$ ***$\log_4 16 = 2$***

(b) $2^5 = 32$ $\quad$ ***$\log_2 32 = 5$***

(c) $p^v = n$ $\quad$ ***$\log_p n = v$***

3.Change back to index form:

eg $\log_2 16 = 4$
so $\quad$ ***$2^4 = 16$***

(a) $\log_3 81 = 4$
$3^4 = 81$

(b) $\log_4 64 = 3$
$4^3 = 64$

(c) $\log_b p = w$
$b^w = p$

4. Find by logical thinking not by using a calculator.

eg $\log_9 81 = x$
so $\quad 9^x = 81$
so $\quad x = 2$

(a) $\log_2 64 = x$

so $\quad 2^x = 64$
$x = 6$

(b) $\log_8 64 = x$

so $\quad 8^x = 64$
$x = 2$

(c) $\log_4 64 = x$
so $\quad 4^x = 64$
$x = 3$

(d) $\log_6 6 = x$
so $\quad 6^x = 6$
$x = 1$

(e) $\log_3 1 = x$
so $\quad 3^x = 1$
$x = 0$

(f) $\log_2 (1/8) = x$
so $\quad 2^x = \frac{1}{8}$
$x = -3$

(g) $\log_2 (1/32) = x$
so $\quad 2^x = \frac{1}{32}$
$x = -5$

(h) $\log_b b = x$
so $\quad b^x = b$
$x = 1$

(i) $\log_b 1 = x$
so $\quad b^x = 1$
$x = 0$

5. The graphs of $y = 2^x$ and $y = \log_2 x$ are very closely related:

Find x, y values for each:

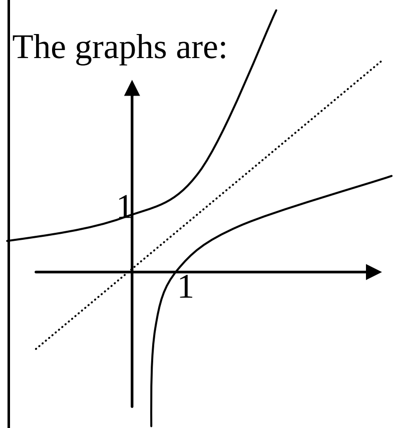
$y = 2^x$

x	y
1	2
2	4
3	8
4	16
-1	.5
-2	.25
-3	.125
-4	.0625
0	1

$y = \log_2 x$

x	y
1	0
2	1
4	2
8	3
16	4
½	-1
¼	-2
1/8	-3
1/32	-4

The graphs are:



Notice that $y = \log_2 x$ is a reflection of $y = 2^x$ in the line $y = x$.

Also notice:
$\log_2(0)$ does not exist

and $\log_2(-b)$ does not exist because the graph does not have any points on the left of the y axis.

Our calculators only have log values to the base 10 and e.
ie $\log_{10} = \log$
and $\log_e = \ln$ (only used in Y13)

Use your calculator to find

1. $\log_{10} 8 =$ **0.9031**

2. $\log_{10} 80 =$ **1.9031**

3. If $\log_{10} x = 2.5$
then $\quad 10^{2.5} = x$
so $\quad x = 316.23$

4. Find x

(a) $\log_{10} x = 3.1$
$10^{3.1} = x$
so $\quad x = 1258.93$

(b) $\log_{10} 3x = 1.5$
$10^{1.5} = 3x$
$3x = 31.62 \qquad x = 10.54$

(c) $\log_{10} x = 3.4$

so $10^{3.4} = x$

$x = 2511.89$

(d) $\log_{10} 5x = 1.6$

so $10^{1.6} = 5x$

$5x = 39.811 \quad x = 7.962$

(e) $10^x = 4.1$

so $\log_{10} 4.1 = x$

$x = 0.6128$

(f) $10^{2x} = 54.2$

so $\log_{10} 54.2 = 2x$

$2x = 1.734$

$x = 0.867$

(g) $10^x = 2.4$

so $\log_{10} 2.4 = x$

$x = 0.3802$

(h) $10^x = 0.7$

so $\log_{10} 0.7 = x$

$x = -0.1549$

(i) $10^{5x} = 4.7$

so $\log_{10} 4.7 = 5x$

$5x = 0.6721$

$x = 0.1344$

(j) $\log_{10}(x + 2) = 0.345$

so $\log_{10} 0.345 = x + 2$

$x + 2 = -0.4622$

$x = -2.4633$

(k) $\log_{10}(x - 4) = 0.3$

so $\log_{10} 0.3 = x - 4$

$x - 4 = -0.5229$

$x = 3.4771$

(l) $10^{2x-4} = 1.3$

so $\log_{10} 1.3 = 2x - 4$

$2x - 4 = 0.1139$

$x = 2.0570$

(m) $10^{4x+1} = 86$

so $\log_{10} 86 = 4x + 1$

$4x + 1 = 1.9345$

$x = 0.2336$

The THREE LOG LAWS.

1. $\log(xy) = \log x + \log y$

This means that when you multiply 2 numbers you add the indices (ie logs)

2 $\log\left(\frac{x}{y}\right) = \log x - \log y$

This means that when you divide 2 numbers you subtract the indices (logs)

3. $\log x^n = n \log x$

This is just an extension of law 1 and becomes more meaningful when we consider:

$\log x^3 = \mathbf{\log x.x.x}$

$\mathbf{= \log x + \log x + \log x}$

$\mathbf{= 3 \log x}$

1.Using these laws, expand these:

eg

$\log a^4b^3 = \mathbf{\log a^4 + \log b^3}$

$\mathbf{= 4 \log a + 3 \log b}$

(a) $\log c^5d^6 = \mathbf{\log c^5 + \log d^6}$

$\mathbf{= 5 \log c + 6\log d}$

(b) $\log \frac{p^5}{v^4}$

$\mathbf{= \log p^5 - \log v^4}$

$\mathbf{= 5\log p - 4 \log v}$

(c) $\log \frac{ab}{c}$

$= \log a + \log b - \log c$

(d) $\log \frac{av}{bw}$ $\mathbf{= \log av - \log bw}$

$\mathbf{= \log a + \log v - \log b - \log w}$

(e) $\log \frac{a c^5}{b^3 n^2} = \log ac^5 - \log b^3n^2$

$= \log a + \log c^5 - (\log b^3 + \log n^2)$

$= \log a + 5\log c - 3\log b - 2\log n$

Combine these into one log function.

eg $\log 3 + \log 5 = \log 15$

(a) $\log 6 - \log 2 = \log 3$

(b) $\log 5 + \log 7 = \mathbf{\log 35}$

(c) $\log 12 - \log 4 = \mathbf{\log 3}$

(d) $\log 7 - \log 8 = \log(7/8)$

(e) $4 \log 2 + 2 \log 3$

$= \mathbf{\log(2^4.3^2) = \log 144}$

The way to find $\log_3 14$ is

let $\mathbf{x = \log_3 14}$

so $\mathbf{3^x = 14}$

then $\mathbf{\log_{10} 3^x = \log_{10} 14}$

so $\mathbf{x \log_{10} 3 = \log_{10} 14}$

so $\mathbf{x = \frac{\log_{10} 14}{\log_{10} 3}}$

Find:

(a) $x = \log_4 15$

$4^x = 15 \quad x \log_{10} 4 = \log_{10} 15$

$x = \log_{10} 15 / \log_{10} 4 = 1.9534$

(b) $x = \log_5 60$

$5^x = 60 \quad x \log_{10} 5 = \log_{10} 60$

$x = \log_{10} 60 / \log_{10} 5 \quad x = 2.544$

(c) $x = \log_7 343$

$x = \log_{10} 343 / \log_{10} 7 \quad x = 3$