UNDERSTANDING LOGARITHMS.

"Log" just means Index or power or exponent.

1. Consider $2^3 = 8$

the index = 3
ie the Log = 3
or in full $\text{Log}_2 8 = 3$

2. Write in log form as shown above.

(a) $4^2 = 16$

(b) $2^5 = 32$

(c) $p^v = n$

3.Change back to index form:
eg $\log_2 16 = 4$
so $2^4 = 16$

(a) $\log_3 81 = 4$

(b) $\log_4 64 = 3$

(c) $\log_b p = w$

4. Find by logical thinking not by using a calculator.
eg $\log_9 81 = x$
so $9^x = 81$
so $x = 2$

(a) $\log_2 64 = x$

(b) $\log_8 64 = x$

(c) $\log_4 64 = x$

(d) $\log_6 6 = x$

(e) $\log_3 1 = x$

(f) $\log_2 (1/8) = x$
so $2^x = \dfrac{1}{8}$
$x = -3$

(g) $\log_2 (1/32) = x$

(h) $\log_b b$

(i) $\log_b 1$

5. The graphs of $y = 2^x$ and $y = \log_2 x$ are very closely related:
Find x, y values for each:

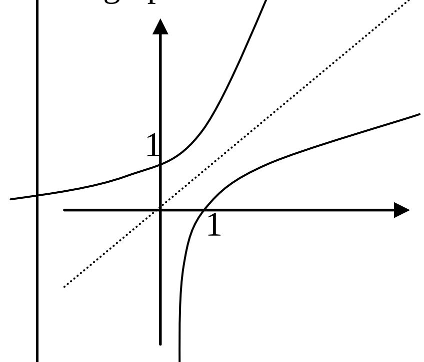
$y = 2^x$

x	y
1	
2	
3	8
4	
-1	
-2	
-3	
-4	
0	

$y = \log_2 x$

x	y
1	
2	
4	
8	3
16	
½	
¼	
1/8	
1/32	

The graphs are:



Notice that $y = \log_2 x$ is a reflection of $y = 2^x$ in the line $y = x$.

Also notice:
$\log_2(0)$ does not exist

and **$\log_2(-b)$ does not exist** because the graph does not have any points on the left of the y axis.

Our calculators only have log values to the base 10 and e.
ie $\log_{10} = \log$
and $\log_e = \ln$ (used later in Y13)

Use your calculator to find

1. $\log_{10} 8 =$

2. $\log_{10} 80 =$

3. If $\log_{10} x = 2.5$
then $10^{2.5} = x$
so $x =$

4. Find x

(a) $\log_{10} x = 3.1$

(b) $\log_{10} 3x = 1.5$

> (c) $\log_{10} x = 3.4$
> so $10^{3.4} = x$

(d) $\log_{10}(5x) = 1.6$

> (e) $10^{x} = 4.1$
> so $\log_{10} 4.1 = x$
> $x =$

(f) $10^{2x} = 54.2$

> (g) $10^{x} = 2.4$
> so $\log_{10} 2.4 = x$
> $x =$

(h) $10^{x} = 0.7$

(i) $10^{(5x)} = 4.7$

(j) $\log_{10}(x + 2) = 0.345$

(k) $\log_{10}(x - 4) = 0.3$

(l) $10^{(2x-4)} = 1.3$

(m) $10^{(4x+1)} = 86$

The THREE LOG LAWS.

1. $\log(xy) = \log x + \log y$

This means that **when you multiply 2 numbers you add the indices** **(ie logs)**

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2. $\log\left(\frac{x}{y}\right) = \log x - \log y$

This means that **when you divide 2 numbers you subtract the indices** **(logs)**

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3. $\log x^n = n \log x$

This is just an extension of law 1 and becomes more meaningful when we consider:

$$\begin{aligned} \log x^3 &= \log x.x.x \\ &= \log x + \log x + \log x \\ &= 3 \log x \end{aligned}$$

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1. Using these laws, expand these:

> eg
> $\log a^4b^3 = \log a^4 + \log b^3$
> $= 4 \log a + 3 \log b$

(a) $\log c^5d^6 =$

(b) $\log \frac{p^5}{v^4}$

(c) $\log \frac{ab}{c}$

(d) $\log \frac{av}{bw} =$

(e) $\log \frac{a\,c^5}{b^3 n^2} =$

> Combine these into one log function.
> eg $\log 3 + \log 5 = \log 15$

(a) $\log 6 - \log 2$

(b) $\log 5 + \log 7$

(c) $\log 12 - \log 4$

(d) $\log 7 - \log 8$

(e) $4 \log 2 + 2 \log 3$

> The way to find $\log_3 14$ is
> let $x = \log_3 14$
> so $3^x = 14$
> then $\log_{10} 3^x = \log_{10} 14$
> so $x \log_{10} 3 = \log_{10} 14$
> so $x = \dfrac{\log_{10} 14}{\log_{10} 3}$

Find:

(a) $\log_4 15$

(b) $\log_5 60$

(c) $\log_7 343$