FURTHER INTERSECTIONS OF LINES and CURVES.

Find the intersections of the following pairs of graphs using algebraic methods.

You will need to use your graphics calculators because the intersections will not always be rational numbers. Be sure to round off numbers appropriately.

1. $y = x^2$

$y = x + 5$

2. $y = x^2 - 20$

$y = x - 4$

3. $x^2 + 4y - 20 = 0$

$y = x - 5$

4. $x^2 + 10y - 60 = 0$

$y = 3x - 4$

5.* If $y = 2x + c$ ***is a tangent to*** $y = x^2 + 3$ ***find the value of c.***

6.* If $y = 3x + c$ ***is a tangent to the parabola*** $y = x^2 + 4y - 20$ ***find the value of c.***

7. $y = \frac{8}{x}$ ***(or*** $xy = 8$***)***

$y = x + 3$

8. $y = \frac{1}{x} + 5$

$y = x - 4$

9. $y = \frac{6}{x} - 9$

$y = 2 - x$

10. $y = 20 - \frac{4}{x}$

$y = 2x - 3$

11.* If $y = x + c$ ***is a tangent to*** $y = 8 - \frac{2}{x}$ ***find c.***

12.* If $y = mx + 2$ ***is a tangent to*** $y = \frac{3}{x}$ ***find the value of m.***

13. $x^2 + y^2 = 25$

$y = x + 2$

14. $x^2 + y^2 = 4$

$y = 2x - 1$

15. $y^2 + x^2 = 49$

$y = 3x + 2$

16* If $y = x + c$ ***is a tangent to*** $x^2 + y^2 = 9$ ***find the possible values of c.***

17*. If $y = x + c$ ***is a tangent to*** $x^2 + y^2 = 4$ ***find the possible values of c.***

18. Find where the line $4y - 7x - 11 = 0$ ***crosses the x and y axes.***

19. Find where the line $y = 7x - 6.38$ ***crosses the x axis.***

20 Find where the line $y = 3x + 7.6295$ ***crosses the x axis.***

SOLUTIONS.

1. $y = x^2 \cap y = x + 5$

$x^2 - x - 5 = 0$

$x = 2.79$ or -1.79

$y = 7.79$ or 3.21

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2. $y = x^2 - 20 \cap y = x - 4$

$x^2 - 20 = x - 4$

$x^2 - x - 16 = 0$

$x = 4.53$ or -3.53

$y = 0.53$ or -7.53

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3. $x^2 + 4y - 20 = 0$
$\cap\ y = x - 5$

$x^2 + 4(x - 5) - 20 = 0$

$x^2 + 4x - 20 - 20 = 0$

$x^2 + 4x - 40 = 0$

$x = 4.63$ or -8.63

$y = -0.37$ or -13.63

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4. $x^2 + 10y - 60 = 0$
$\cap\ y = 3x - 4$

$x^2 + 10(3x - 4) - 60 = 0$

$x^2 + 30x - 40 - 60 = 0$

$x^2 + 30x - 100 = 0$

$x = 3.03$ or -33.03

$y = 5.09$ or 95.09

5.* If $y = 2x + c$ is a tangent to $y = x^2 + 3$ find the value of c.

$x^2 + 3 = 2x + c$

$x^2 - 2x + (3 - c) = 0$

To be a tangent $\Delta = 0$ so

$4 - 4(3 - c) = 0$

$4 - 12 + 4c = 0$

$4c = 8$ so $c = 2$

6.* If $y = 3x + c$ is a tangent to the parabola $y = x^2 + 4y - 20$ find the value of c.

$x^2 + 4(3x + c) - 20 = 0$

$x^2 + 12x + (4c - 20) = 0$

To be a tangent $\Delta = 0$ so

$144 - 4(4c - 20) = 0$

$144 - 16c + 80 = 0$

$64 = 16c$ so $c = 4$

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7. $xy = 8 \cap y = x + 3$

$x(x + 3) = 8$

$x^2 + 3x - 8 = 0$

$x = 1.702$ or -4.702

$y = 4.702$ or -1.702

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8. $y = \frac{1}{x} + 5 \cap y = x - 4$

$x - 4 = \frac{1}{x} + 5$

$x^2 - 4x = 1 + 5x$

$x^2 - 9x - 1 = 0$

$x = 9.11$ or -0.11

$y = 5.11$ or -4.11

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9. $y = \frac{6}{x} - 9 \cap y = 2 - x$

$2 - x = \frac{6}{x} - 9$

$2x - x^2 = 6 - 9x$

$0 = x^2 - 11x + 6$

$x = 10.42$ or 0.576

$y = -8.42$ or 1.424

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10. $y = 20 - \frac{4}{x} \cap y = 2x - 3$

$2x - 3 = 20 - \frac{4}{x}$

$2x^2 - 3x = 20x - 4$

$2x^2 - 23x + 4 = 0$

$x = 11.32$ or 0.177

$y = 19.64$ or -2.646

11.* If $y = x + c$ is a tangent to $y = 8 - \frac{2}{x}$ find c.

$x + c = 8 - \frac{2}{x}$

$x^2 + cx = 8x - 2$

$x^2 + (c - 8)x + 2 = 0$

To be a tangent $\Delta = 0$ so

$(c - 8)^2 - 4 \times 2 = 0$

$(c - 8)^2 = 8$

$c - 8 = 2.828$ or -2.828

$c = 10.828$ or 5.172

12.* If $y = mx + 2$ is a tangent to $y = \frac{3}{x}$ find the value of m.

$mx + 2 = \frac{3}{x}$

$mx^2 + 2x - 3 = 0$

To be a tangent $\Delta = 0$ so

$4 - 4m(-3) = 0$

$4 = 12m$

$m = \frac{1}{3}$

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13. $x^2 + y^2 = 25$

$\cap$ $y = x + 2$

$x^2 + (x + 2)^2 = 25$
$x^2 + x^2 + 4x + 4 - 25 = 0$
$2x^2 + 4x - 21 = 0$
$x = 2.39$ or -4.39
$y = 4.39$ or -2.39

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14. $x^2 + y^2 = 4$

$\cap$ $y = 2x - 1$

$x^2 + (2x - 1)^2 = 4$
$x^2 + 4x^2 - 4x + 1 - 4 = 0$
$5x^2 - 4x - 3 = 0$
$x = 1.272$ or -0.472
$y = 1.544$ or -1.944

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15. $y^2 + x^2 = 49$

$\cap$ $y = 3x + 2$

$x^2 + (3x + 2)^2 = 49$
$x^2 + 9x^2 + 12x + 4 - 49 = 0$
$10x^2 + 12x - 45 = 0$
$x = 1.605$ or -2.805
$y = 6.815$ or -3.61

16* If $y = x + c$ is a tangent to $x^2 + y^2 = 9$ find the possible values of c.

$x^2 + (x + c)^2 = 9$
$x^2 + x^2 + 2cx + c^2 - 9 = 0$
$2x^2 + 2cx + (c^2 - 9) = 0$
To be a tangent $\Delta = 0$ so
$4c^2 - 4 \times 2 \times (c^2 - 9) = 0$
$4c^2 - 8c^2 + 72 = 0$
$72 = 4c^2$
$c^2 = 18$
$c = 4.243$ or -4.243

17*. If $y = x + c$ is a tangent to $x^2 + y^2 = 4$ find the possible values of c.

$x^2 + (x + c)^2 = 4$
$x^2 + x^2 + 2cx + c^2 - 4 = 0$
$2x^2 + 2cx + (c^2 - 4) = 0$
To be a tangent $\Delta = 0$ so
$4c^2 - 4 \times 2 \times (c^2 - 4) = 0$
$4c^2 - 8c^2 + 32 = 0$
$32 = 4c^2$
$c^2 = 8$
$c = 2.828$ or -2.828

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18. Find where the line $4y - 7x - 11 = 0$ crosses the x and y axes.

Crosses x axis if $y = 0$ so
$-7x - 11 = 0$
$-11 = 7x$
$x = -1.57$

point is $(-1.57, 0)$

Crosses y axis if $x = 0$ so
$4y - 11 = 0$
$y = 2.75$

point is $(0, 2.75)$

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19. Find where the line $y = 7x - 6.38$ crosses the x axis.

Crosses x axis if $y = 0$ so

$7x - 6.38 = 0$
$7x = 6.38$
$x = 0.911$

point is $(0.911, 0)$

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20 Find where the line $y = 3x + 7.6295$ crosses the x axis.

Crosses x axis if $y = 0$ so

$3x - 7.6295 = 0$
$3x = 7.6295$
$x = 2.543$

point is $(2.543, 0)$