

Y12 : PRACTICE ASSESSMENT B. ACHIEVEMENT LEVEL ONLY.

Algebra.

1. Expand:

$$(a) (5x-4)^2 = (5x-4)(5x-4) \\ = 25x^2 - 40x + 16$$

$$(b) (x-2)(x+3)(x-5) \\ = (x-2)(x^2 - 2x - 15) \\ = x^3 - 2x^2 - 15x - 2x^2 + 4x + 30 \\ = x^3 - 4x^2 - 11x + 30$$

2. Simplify fully:

$$(a) \sqrt{9c^{16}} = 3c^8$$

$$(b) (27a^3b^{12})^{2/3} = (3ab^4)^2 = 9a^2b^8$$

3. Combine into one

log function:

$$3\log a - 2\log b = \log_3 a^3 - \log_3 b^2 \\ = \log_3 \left(\frac{a^3}{b^2} \right)$$

4. Simplify fully:

$$\frac{(x+3) \cdot 3}{(x+3)(x+2)} - \frac{2(x+2)}{(x+3)(x+2)} \\ = \frac{3x+9-2x-4}{(x+3)(x+2)} \\ = \frac{x+5}{(x+3)(x+2)}$$

5. Solve:

$$(a) 9(x+2) = 5(x-3) \\ 9x + 18 = 5x - 15 \\ 4x = -33 \\ x = -\frac{33}{4}$$

$$(b) 5x^2 + 2x - 3 = 0 \\ (5x-3)(x+1) = 0 \\ x = \frac{3}{5}, -1$$

$$(c) -3x > 12$$

$$x < -4$$

$$(d) \log_4 x = 4$$

$$4^4 = x \quad \therefore x = 256$$

$$(e) \log_b 64 = 2$$

$$b^2 = 64 \quad \therefore b = 8$$

6. Solve:

$$(a) (x^2-9)(x^2-4) = 0 \\ (x+3)(x-3)(x+2)(x-2) = 0 \\ x = -3, 3, -2, 2$$

$$(b) \frac{3(x-4)}{4} - \frac{(x-2)}{2} < 2 \quad \therefore 4 \left[\frac{3(x-4)}{4} - \frac{(x-2)}{2} \right] < 8 \\ 3x-12 - 2(x-2) < 8 \\ 3x-12 - 2x+4 < 8 \\ x-8 < 8 \\ x < 16$$

7. Rearrange the formula $V = \pi r^2 h$ to make r the subject.

$$\pi r^2 h = V \\ r^2 = \frac{V}{\pi h} \\ r = \pm \sqrt{\frac{V}{\pi h}}$$

Calculus.

1. Find the gradient of $y = x^3$ at $x = 2$

$$y' = 3x^2$$

$$\text{sub } x = 2 \quad y' = 12$$

2. Find the x value of the point on the curve $y = x^2 + 2x$ where the gradient equals 8.

$$\begin{aligned} y' &= 2x + 2 = 8 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

3. Find the x value of the points on the curve $y = 2x^3 - 6x$ where the gradient is zero.

$$\begin{aligned} y' &= 6x^2 - 6 = 0 \\ 6(x^2 - 1) &= 0 \\ 6(x+1)(x-1) &= 0 \\ x &= -1, 1 \end{aligned}$$

4. Find y if $y' = 5x^3 + 7x^2 + 1$

$$y = \frac{5x^4}{4} + \frac{7x^3}{3} + x + C$$

5. Find the equation of the curve given that $f'(x) = 12x^2 - 2$ and the point $(3, 4)$ is on the curve.

$$\begin{aligned} y &= 4x^3 - 2x + C \\ \text{Subs } 4 &= 108 - 6 + C \\ 4 &= 102 + C \quad \text{so } C = -98 \\ y &= 4x^3 - 2x - 98 \end{aligned}$$

6. If $y' = x(6-x)$ find y if $x=1, y=10$

$$\begin{aligned} y' &= 6x - x^2 \\ \text{so } y &= 3x^2 - \frac{x^3}{3} + C \end{aligned}$$

$$\begin{aligned} \text{subs } 10 &= 3 - \frac{1}{3} + C \\ 10 &= 2\frac{2}{3} + C \\ 7\frac{1}{3} &= C \\ y &= 3x^2 - \frac{x^3}{3} + 7\frac{1}{3} \end{aligned}$$

7. Given that $f'(x) = 4x + 3x^2$

find $f(x)$ if $f(1) = 2$

$$\begin{aligned} y' &= 4x + 3x^2 \\ y &= 2x^2 + x^3 + C \end{aligned}$$

$$\begin{aligned} \text{Subs } 2 &= 2 + 1 + C \\ -1 &= C \\ y &= 2x^2 + x^3 - 1 \end{aligned}$$