Year 12 mathematics: INTERSECTIONS OF LINES, PARABOLAS AND CIRCLES: ANSWERS.

1. $y = x^2$
$y = x + 6$
$x^2 = x + 6$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2)=0$
$x = 3$ and $x = -2$
when $x = 3, y = 9$
when $x = -2, y = 4$
intersect at $(3, 9)$ & $(-2, 4)$

2. $y = x^2 - 3x - 4$
$y = x - 7$
$x - 7 = x^2 - 3x - 4$
$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
$x = 3$ and $x = 1$
when $x = 3, y = -4$
when $x = 1, y = -6$
intersect at $(3, -4)$ & $(1, -6)$

3. $y = x^2 - 4$
$y = 2x - 5$
$x^2 - 4 = 2x - 5$
$x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0$
$x = 1$
when $x = 1, y = 2\times1-5 = -3$
tangent at $(1, -3)$

4. $y = x^2$
$y = x - 4$
$x^2 = x - 4$
$x^2 - x + 4 = 0$
no solution, no intersection

5. $y = x^2 + 2x - 8$
$y = 2x - 4$
$x^2 + 2x - 8 = 2x - 4$
$x^2 - 4 = 0$
$(x + 2)(x - 2) = 0$
$x = -2$ and $x = 2$
when $x = -2, y = -8$
when $x = 2, y = 0$
intersect at $(2, 0)$ & $(-2, -8)$

6. $y = x^2 + x - 2$
$y = 2x + p$
$x^2 + x - 2 = 2x + p$
$x^2 - x - 2 - p = 0$
At point of tangency,
discriminant $\Delta = 0$
$(-1)^2 - 4 \times 1 \times (-2 - p) = 0$
$1 + 8 + 4p = 0$
$p = -2.25$

7. $y = \frac{1}{x} \leftrightarrow xy = 1$
$y = 2 - x$
$x(2 - x) = 1$
$2x - x^2 = 1$
$x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0$
$x = 1$
when $x = 1, y = 1$
tangent at $(1,1)$

8. $y = \frac{1}{x} \leftrightarrow xy = 1$
$y = 2x - 1$
$x(2x - 1) = 1$
$2x^2 - x - 1 = 0$
$x = 1$ and $x = -0.5$
when $x = 1, y = 1$
when $x = -0.5, y = -2$
intersect $(1,1)$ & $(-0.5, -2)$

9. $y = \frac{6}{x} \leftrightarrow xy = 6$
$y = 7 - x$
$x(7 - x) = 6$
$7x - x^2 = 6$
$x^2 - 7x + 6 = 0$
$(x - 6)(x - 1) = 0$
$x = 6$ and $x = 1$
when $x = 6, y = 1$
when $x = 1, y = 6$
intersect at $(1, 6)$ and $(6, 1)$

10. $y = \frac{-4}{x} \leftrightarrow xy = -4$
$y = x - 5$
$x(x - 5) = -4$
$x^2 - 5x + 4 = 0$
$(x - 1)(x - 4) = 0$
$x = 1$ and $x = 4$
when $x = 1, y = -4$
when $x = 4, y = -1$
intersect at $(1, -4)$ & $(4, -1)$

11. $y = \frac{4}{x} \leftrightarrow xy = 4$
$y = b - x$
$x(b - x) = 4$
$bx - x^2 = 4$
$x^2 - bx + 4 = 0$
At point of tangency,
discriminant $\Delta = 0$
$(-b)^2 - 4 \times 1 \times 4 = 0$
$b^2 - 16 = 0$
$b = -4$ and $b = 4$
tangents when $b=-4$ & $b=4$.

12. $y = \frac{2}{x} \leftrightarrow xy = 2$
$y = mx + 8$
$x(mx + 8) = 2$
$mx^2 + 8x = 2$
$mx^2 + 8x - 2 = 0$
At point of tangency,
discriminant $\Delta = 0$
$8^2 - 4 \times m \times -2 = 0$
$64 + 8m = 0$
$m = -8$

13. $x^2 + y^2 = 25$
$y = x - 1$
$x^2 + (x - 1)^2 = 25$
$x^2 + x^2 - 2x + 1 = 25$
$2x^2 - 2x - 24 = 0$
$x = 4$ and $x = -3$
when $x = 4, y = 3$
when $x = -3, y = -4$
intersect at $(4, 3)$ & $(-3, -4)$

14. $x^2 + y^2 = 25$
$y = 2x - 2$
$x^2 + (2x - 2)^2 = 25$
$x^2 + 4x^2 - 8x + 4 = 25$
$5x^2 - 8x - 21 = 0$
$x = 3$ and $x = -1.4$
when $x = 3, y = 4$
when $x = -1.4, y = -4.8$
intersect at $(3, 4)$ and
$(-1.4, -4.8)$

15. $x^2 + y^2 = 5$
$y = x + 1$
$x^2 + (x + 1)^2 = 5$
$x^2 + x^2 + 2x + 1 = 5$
$2x^2 + 2x - 4 = 0$
$x = 1$ and $x = -2$
when $x = 1, y = 2$
when $x = -2, y = -1$
intersect at $(1, 2)$ & $(-2, -1)$

16. $x^2 + y^2 = 13$
$y = x + 1$
$x^2 + (x + 1)^2 = 13$
$x^2 + x^2 + 2x + 1 = 13$
$2x^2 + 2x - 12 = 0$
$x = 2$ and $x = -3$
when $x = 2, y = 3$
when $x = -3, y = -2$
intersect at $(2, 3)$ & $(-3, -2)$

17. $x^2 + y^2 = 10$
$y = 3x$
$x^2 + (3x)^2 = 10$
$x^2 + 9x^2 = 10$
$10x^2 = 10$
$x = 1$ and $x = -1$
when $x = 1, y = 3$
when $x = -1, y = -3$
intersect at $(1, 3)$ & $(-1, -3)$

18. $x^2 + y^2 = 8$
$y = x + 4$
$x^2 + (x + 4)^2 = 8$
$x^2 + x^2 + 8x + 16 = 8$
$2x^2 + 8x + 8 = 0$
$x = -2, y = 2$
tangent at $(-2, 2)$

19. $x^2 + y^2 = 2$
$y = x + p$
$x^2 + (x + p)^2 = 2$
$x^2 + x^2 + 2px + p^2 - 2 = 0$
$2x^2 + 2px + p^2 - 2 = 0$
At point of tangency,
discriminant $\Delta = 0$
$(2p)^2 - 4\times 2\times(p^2 - 2) = 0$
$4p^2 - 8p^2 + 16 = 0$
$-4p^2 = -16$
$p = 2$ and $p = -2$

20. $x^2 + y^2 = 2$
$y = \frac{1}{x}$
$x^2 + (\frac{1}{x})^2 = 2$
$x^4 + 1 = 2x^2$
$x^4 - 2x^2 + 1 = 0$
$x^2 = 1$
$x = 1$ and $x = -1$
when $x = 1, y = 1$
when $x = -1, y = -1$
Intersect at $(1,1)$ & $(-1, -1)$