FUNDAMENTALS of INDICES.

General Objectives:

1. To know that 3^4 **means** $3\times3\times3\times3$

2(a). To understand WHY $b^3 \times b^4 = b^7$ (**NOT just because it is a "rule**")

i.e. $b^3 \times b^4 = b.b.b \;.\; b.b.b.b. = b^7$

(b) so that $b^n \times b^p = b^{n+p}$

3.(a) To understand *WHY* $\dfrac{b^5}{b^3} = b^2$

i.e. $\dfrac{b^5}{b^3} = \dfrac{b.b.b.b.b.}{b.b.b.} = b^2$

(b) so that $\dfrac{b^n}{b^p} = b^{n-p}$

THE FOLLOWING CONCEPTS ARE DIFFICULT TO FULLY GRASP BUT THEY ARE ESSENTIAL FOR PROPER UNDERSTANDING.

4. To understand WHY $b^0 = 1$

i.e. $\dfrac{b^2}{b^2} = \dfrac{b.b}{b.b} = 1$ using the idea in 3(*a*)

$\dfrac{b^2}{b^2} = b^{2-2} = b^0$ using the idea in 3(b)

} so ***b^0 must be 1***

5. To understand WHY $b^{-2} = \dfrac{1}{b^2}$ (see note 7. below)

i.e. $\dfrac{b^3}{b^5} = \dfrac{b.b.b}{b.b.b.b.b} = \dfrac{1}{b^2}$ using the idea in 3(*a*)

$\dfrac{b^3}{b^5} = b^{3-5} = b^{-2}$ using the idea in 3(b)

} ***so b^{-2} must be $\dfrac{1}{b^2}$***

6. To understand WHY $b^{½} = \sqrt{b}$

i.e. $9^{½} \times 9^{½} = 9^{½+½} = 9^1 = 9$

but $3 \times 3 = 9$

} **so $9^{½} = \sqrt{9} = 3$**

7. I believe the logic in note 5 is most important but only the very best students can reproduce it later on request.
   We must not just rely on the so called "logical" pattern as follows:

$10^3 = 1000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$
$10^{-1} = \frac{1}{10}$
$10^{-2} = \frac{1}{100}$
etc

…..because, to some students, the following seems just as "logical" :

$10^3 = 1000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$
$10^{-1} = -10$!!!
$10^{-2} = -10$!!!
$10^{-3} = -100$!!!

Teaching notes:

1. Practice is needed just multiplying out 2^4, 3^2, 7^2, 5^3 etc.
   Also teach the **power function** on the calculator.

2. The idea that $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$ is difficult for MOST people but important.

Standard Form should not become a meaningless application of RULES. It should be UNDERSTOOD!

i.e. you should know that $0.0012 = \frac{1.2}{10^3}$ **or** 1.2×10^{-3}

3. $b^0 = 7^0 = 88^0 = 1$ etc seems to be REMEMBERED but is it UNDERSTOOD?

4. Basic examples:

(i) $\frac{c^8}{c^6} = \frac{c^2}{1}$ *or* c^2

(ii) $\frac{d^3}{d^5} = \frac{1}{d^2}$ *(BUT discourage* d^{-2} *)*

(iii) $\frac{a^5 \times b^2}{a^3 \times b^9} = \frac{a^2}{b^7}$ covering all possible cancelling combinations.

5. **Also** $\boldsymbol{(b^4)^3 = b^4 . b^4 . b^4 = b^{12}}$

6 It would be sad if this topic were reduced to the mere application of "rules" without some understanding of how the "rules" are derived.

ie we <u>don't</u> want students just memorising RULES like :

$$b^n \times b^p = b^{n+p}$$

and $\frac{b^n}{b^p} = b^{n-p}$

and $(b^n)^p = b^{np}$