

## SOLVING EQUATIONS AND INEQUALITIES

(some of which have other unknowns).

In solving linear equations and inequalities,  
**there are only 4 processes which we need to apply:**

We can **ADD** something to **both sides**,  
**SUBTRACT** something from **both sides**,  
**MULTIPLY** **both sides** by something or  
**DIVIDE** **both sides** by something.

Find  $x$  in the four fundamental cases:

$$1(a) \quad x + 8 = 12$$

$$(b) \quad x + k = w$$

$$2(a) \quad x - 6 > 18$$

$$(b) \quad x - k > w$$

$$3(a) \quad 4x = 12$$

$$(b) \quad kx = w$$

$$4(a) \quad \frac{x}{4} < 8$$

$$(b) \quad \frac{x}{k} < w$$

Use **combinations** of the above four techniques to solve the following equations or inequalities to find the  $x$  values.

$$5(a) \quad 5x + 3 = 11$$

$$(b) \quad px + v = m$$

$$6(a) \quad \frac{x}{2} - 4 \leq 3$$

$$(b) \quad \frac{x-t}{c} \leq v$$

$$7(a) \quad 7x - 4 = 3x + 11$$

$$(b) \quad cx - b = nx + p$$

$$8(a) \quad 5(x + 4) \geq 3(x - 9)$$

$$(b) \quad p(x + c) \geq v(x - d)$$

$$9(a) \quad 5(x + 3) + 2(x + 4) = 2(x + 15)$$

$$(b) \quad c(x + p) + b(x + v) = d(x + a)$$

$$10(a) \quad 7(x - 2) - 3(x - 4) < 2(x - 5)$$

$$(b) \quad a(x - b) - c(x - d) < e(x - f)$$

$$11(a) \quad \frac{x}{2} + \frac{5}{3} > 7$$

$$(b) \quad \frac{x}{5} + \frac{v}{2} > p$$

$$12(a) \quad \frac{15}{(x - 2)} = 3$$

$$(b) \quad \frac{p}{(x - b)} = c$$

$$13(a) \quad \frac{12}{(x + 4)} = \frac{4}{x}$$

$$(b) \quad \frac{b}{(x + v)} = \frac{c}{x}$$

$$14(a) \quad \frac{7}{(x + 2)} = \frac{9}{(x - 3)}$$

$$(b) \quad \frac{b}{(x + v)} = \frac{c}{(x - m)}$$

## MODEL SOLUTIONS

Find  $x$  in the four fundamental cases:

$$1(a) \quad x + 8 = 12 \text{ (subtract 8 from b.s.)}$$

$$x = 4$$

$$(b) \quad x + k = w \text{ (subtract } k \text{ from b.s.)}$$

$$x = w - k$$

$$2(a) \quad x - 6 > 18 \text{ (add 6 to b.s.)}$$

$$x > 24$$

$$(b) \quad x - k > w \text{ (add } k \text{ to b.s.)}$$

$$x > w + k$$

$$3(a) \quad 4x = 12 \text{ (divide b.s. by 4)}$$

$$x = 3$$

$$(b) \quad kx = w \text{ (divide b.s. by } k)$$

$$x = \frac{w}{k}$$

$$4(a) \quad \frac{x}{4} < 8 \text{ (multiply b.s. by 4)}$$

$$x < 32$$

$$(b) \quad \frac{x}{k} < w \text{ (multiply b.s. by } k)$$

$$x < kw$$

Use combinations of the above four techniques to solve the following equations or inequalities to find the  $x$  values.

$$5(a) \quad 5x + 3 = 11$$

$$5x = 8$$

$$x = \frac{8}{5}$$

$$(b) \quad px + v = m$$

$$px = m - v$$

$$x = \frac{m - v}{p}$$

$$6(a) \quad \frac{x}{2} - 4 \leq 3$$

$$\frac{x}{2} \leq 7$$

$$x \leq 14$$

$$(b) \quad \frac{x}{c} - t \leq v$$

$$\frac{x}{c} \leq v + t$$

$$x \leq c(v + t)$$

$$7(a) \quad 7x - 4 = 3x + 11$$

$$4x - 4 = 11$$

$$4x = 15$$

$$x = \frac{15}{4}$$

$$(b) \quad cx - b = nx + p$$

$$(c - n)x - b = p$$

$$(c - n)x = p + b$$

$$x = \frac{p + b}{(c - n)}$$

$$8(a) \quad 5(x + 4) \geq 3(x - 9)$$

$$5x + 20 \geq 3x - 27$$

$$2x + 20 \geq -27$$

$$2x \geq -47$$

$$x \geq \frac{-47}{2}$$

$$(b) \quad p(x + c) \geq v(x - d)$$

$$px + pc \geq vx - vd$$

$$(p - v)x + pc \geq -vd$$

$$(p - v)x \geq -vd - pc$$

$$x \geq \frac{-vd - pc}{(p - v)}$$

$$9(a) \quad 5(x+3) + 2(x+4) = 2(x+15)$$

$$5x + 15 + 2x + 8 = 2x + 30$$

$$5x + 23 = 30$$

$$5x = 7$$

$$x = \frac{7}{5}$$

$$(b) \quad c(x+p) + b(x+v) = d(x+a)$$

$$cx + cp + bx + bv = dx + da$$

$$cx + bx - dx = da - cp - bv$$

$$x(c+b-d) = da - cp - bv$$

$$x = \frac{da - cp - bv}{(c+b-d)}$$

$$10(a) \quad 7(x-2) - 3(x-4) < 2(x-5)$$

$$7x - 14 - 3x + 12 < 2x - 10$$

$$7x - 3x - 2x < -10 + 14 - 12$$

$$2x < -8$$

$$x < -4$$

$$(b) \quad a(x-b) - c(x-d) < e(x-f)$$

$$ax - ab - cx + cd < ex - ef$$

$$ax - cx - ex < -ef + ab - cd$$

$$x(a-c-e) < -ef + ab - cd$$

$$x < \frac{-ef + ab - cd}{(a-c-e)}$$

$$11(a) \quad \frac{x}{2} + \frac{5}{3} > 7$$

$$6\left(\frac{x}{2} + \frac{5}{3}\right) > 6 \times 7$$

$$3x + 10 > 42$$

$$3x > 32$$

$$x > \frac{32}{3}$$

$$(b) \quad \frac{x}{5} + \frac{v}{2} > p$$

$$10\left(\frac{x}{5} + \frac{v}{2}\right) > 10p$$

$$2x + 5v > 10p$$

$$2x > 10p - 5v$$

$$x > \frac{10p - 5v}{2}$$

$$12(a) \quad \frac{15}{(x-2)} = 3$$

$$\frac{15}{3} = x - 2$$

$$\frac{5}{7} = x - 2$$

$$7 = x$$

$$(b) \quad \frac{p}{(x-b)} = c$$

$$\frac{p}{c} = x - b$$

$$\frac{p}{c} + b = x$$

$$13(a) \quad \frac{12}{(x+4)} = \frac{4}{x}$$

$$12x = 4x + 16$$

$$8x = 16$$

$$x = 2$$

$$(b) \quad \frac{b}{(x+v)} = \frac{c}{x}$$

$$bx = cx + cv$$

$$bx - cx = cv$$

$$x(b-c) = cv$$

$$x = \frac{cv}{(b-c)}$$

$$14(a) \quad \frac{7}{(x+2)} = \frac{9}{(x-3)}$$

$$7x - 21 = 9x + 18$$

$$-21 = 2x + 18$$

$$-39 = 2x$$

$$\frac{-39}{2} = x$$

$$(b) \quad \frac{b}{(x+v)} = \frac{c}{(x-m)}$$

$$bx - bm = cx + cv$$

$$bx - cx = cv + bm$$

$$x(b-c) = cv + bm$$

$$x = \frac{cv + bm}{(b-c)}$$