5a PARALLEL CALCULUS QUESTIONS FROM NCEA EXAMS.

ACHIEVED LEVEL

1a ***If $f(x) = 3x^2 - 5x + 3$ find the gradient at $x = \frac{1}{2}$*** $\frac{dy}{dx} = 6x - 5$ sub $x = \frac{1}{2}$ grad $\frac{dy}{dx} = 6 \times \frac{1}{2} - 5$ $= 3 - 5$ $= -2$	1b ***If $\frac{dy}{dx} = 3x^2 + 6x + 2$ find the equation for y given that when $x = 1$, $y = 5$*** $y = x^3 + 3x^2 + 2x + c$ sub $(1, 5)$ $5 = 1 + 3 + 2 + c$ $5 = 6 + c$ so $c = -1$ equ'n $y = x^3 + 3x^2 + 2x - 1$
2a ***Find the x coordinate where the gradient of $y = 4x^2 - 12x + 5$ equals 2*** $\frac{dy}{dx} = 8x - 12 = 2$ $8x = 14$ $x = \frac{14}{8}$ or $\frac{7}{4}$	2b ***The pressure P in a tube at t secs is given by $P = t^3 + t^2 + 5t$. Find the rate of increase of pressure when $t = 4$ secs*** $\frac{dP}{dt} = 3t^2 + 2t + 5$ sub $t = 4$ $\frac{dP}{dt} = 3 \times 16 + 2 \times 4 + 5$ $= 61$
3a ***Find the x values when the gradient of $y = 2x^3 - 3x^2 - 12x$ is zero.*** $\frac{dy}{dx} = 6x^2 - 6x - 12 = 0$ $6(x^2 - x - 2) = 0$ $6(x - 2)(x + 1) = 0$ $x = 2, -1$	3b ***Find the equation of the curve that goes through (0, 0) and has a gradient of $\frac{dy}{dx} = x^3 - x^2 + x - 1$*** $y = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x + c$ sub $(0,0)$ so $c = 0$ $y = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x$

MERIT LEVEL

1c ***A flare is fired from a boat. The height of the flare is given by $H = 80t - 5t^2 + 3$. Find the maximum height of the flare.***

$\frac{dH}{dt} = 80 - 10t = 0$ at max

$t = 8$ sec

$H_{max} = 80 \times 8 - 5 \times 64 + 3$

$= 323$ m

1d ***Find the coordinates of the max/min points on the curve $y = x^3 - 2x^2 - 4x + 3$ and distinguish between them.***

$y' = 3x^2 - 4x - 4 = 0$ at max/min

$(3x + 2)(x - 2) = 0$

$x = -\frac{2}{3}, \; 2$

cubic is like this

So max is $(-\frac{2}{3}, 4.48)$

and min is $(2, -5)$

2c ***A stone is dropped into a pool of water and a circular ripple is formed. The area of the ripple is $A = \pi r^2$. Find the rate of increase in the area of the ripple, with respect to r, when the area is 64π m^2***

Rate of inc is $\frac{dA}{dr} = 2\pi \times r$

If $A = 64\pi$

then $\pi r^2 = 64\pi$

$r^2 = 64$

$r = 8$ (not -8)

So rate $\frac{dA}{dr} = 2\pi \times 8 = 16\pi$

2d ***The gradient of a parabola is given by $\frac{dy}{dx} = 2x - 10$ and 6 is the minimum value of the curve. Find the equation of the curve.***

Min is when $\frac{dy}{dx} = 2x - 10 = 0$

so $x = 5$

$y = x^2 - 10x + c$

goes through $(5, 6)$

$6 = 25 - 50 + c$

so $c = 31$

curve is $y = x^2 - 10x + 31$

3c ***Find the equation of the tangent to the curve $y = 0.5x^2 - 4x + 3$ at $x = 2$***

grad $y' = x - 4$

Sub $x = 2$ grad $= -2$

If $x = 2$ $y = 2 - 8 + 3 = -3$

Tan is like $y = mx + c$

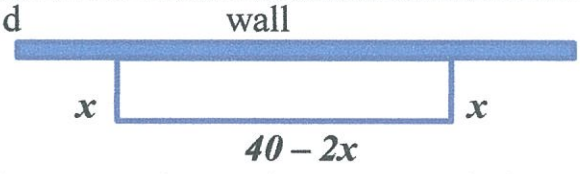
subs $-3 = -2 \times 2 + c$

$-3 = -4 + c$

$1 = c$

tan is $y = -2x + 1$

3d



A rectangular enclosure is made from 40 metres of fence using a wall as one side. Use calculus to find the maximum area of the enclosure.

Area $A = x(40 - 2x) = 40x - 2x^2$

$\frac{dA}{dx} = 40 - 4x = 0$ for max

so $x = 10$

Max $A = 10 \times 20 = 200$ m^2