Excellence Revision 2014

1. Find the range of values of p for which the equation $x + 4 = 2\sqrt{(x + p)}$ has two distinct real solutions.

$(x+4)^2 = 4(x+p)$

$x^2 + 8x + 16 = 4x + 4p$

$x^2 + 4x + (16-4p) = 0$

This has 2 sols if $\Delta > 0$

$4^2 - 4 \times 1 \times (16-4p) > 0$

$16 - 64 + 16p > 0$

$-48 + 16p > 0$

$16p > 48$

$p > 3$

2. Find the equation whose roots are 4 times those of $x^2 + 6x + 12 = 0$

(Very hard – should be Y 13)

If roots are α, β

$(x-\alpha)(x-\beta) = 0$

$x^2 - \alpha x - \beta x + \alpha\beta = 0$

$\begin{cases} x^2 - (\alpha+\beta)x + \alpha\beta = 0 \\ x^2 + 6x + 12 = 0 \end{cases}$

so $\alpha + \beta = -6$ and $\alpha\beta = 12$

If roots are $4\alpha, 4\beta$

equin $(x - 4\alpha)(x - 4\beta) = 0$

$x^2 - 4\alpha x - 4\beta x + 16\alpha\beta = 0$

$x^2 - 4(\alpha+\beta)x + 16\alpha\beta = 0$

Subs $\alpha + \beta = -6$, $\alpha\beta = 12$

$x^2 - 4 \times {-6}\, x + 16 \times 12 = 0$

$x^2 + 24x + 192 = 0$

3. Solve the following equation for x in terms of k where $k > 0$

($\ln = \log$)

$$\ln(3x - 2) - \ln(x - 5) = 2\ln(k)$$

$\log\left(\frac{3x-2}{x-5}\right) = \log K^2$

so $\frac{3x-2}{x-5} = K^2$

$3x - 2 = K^2 x - 5K^2$

$3x - K^2 x = 2 - 5K^2$

$x(3 - K^2) = 2 - 5K^2$

$x = \frac{2 - 5K^2}{3 - K^2}$

4. Solve the following equation to find an expression for ***x*** in

terms of ***p***: $\log_3(x-p)=2$.

$$x - p = 3^2$$
$$x - p = 9$$
$$x = 9 + p$$

5. Solve the equation for ***x*** in terms of ***p*** : $3^{(x-p)} = 2^{(x+p)}$

$$\log 3^{x-p} = \log 2^{x+p}$$
$$(x-p)\log 3 = (x+p)\log 2$$
$$x\log 3 - p\log 3 = x\log 2 + p\log 2$$
$$x\log 3 - x\log 2 = p\log 2 + p\log 3$$
$$x(\log 3 - \log 2) = p(\log 2 + \log 3)$$
$$x = \frac{p(\log 2 + \log 3)}{(\log 3 - \log 2)}$$

6 Solve for ***x*** *in terms of* ***t***:

log(x + 4) – log(x) = log(t)

$$\log\left(\frac{x+4}{x}\right) = \log t$$
$$\frac{x+4}{x} = t$$
$$x + 4 = xt$$
$$4 = xt - x$$
$$4 = x(t-1)$$
$$\frac{4}{(t-1)} = x$$

7. Solve the following equation to find an expression for x in terms of b:
$b\sqrt{(x - b)} = \sqrt{(x + 2b)}$ (There is no need to check the validity of your answer.)

$$b^2(x-b) = x + 2b$$
$$b^2x - b^3 = x + 2b$$
$$b^2x - x = b^3 + 2b$$
$$x(b^2-1) = b^3 + 2b$$
$$x = \frac{b^3 + 2b}{b^2-1}$$

8. Solve the following equation for x in terms of c :

$$2^{(x+3)} = 3^{cx}$$

$$\log 2^{x+3} = \log 3^{cx}$$
$$(x+3)\log 2 = cx \log 3$$
$$x \log 2 + 3\log 2 = cx \log 3$$
$$3 \log 2 = cx \log 3 - x \log 2$$
$$3 \log 2 = x(c \log 3 - \log 2)$$
$$\frac{3 \log 2}{(c \log 3 - \log 2)} = x$$

9. Solve for x in terms of a and b

$$a^{(x+2)} = b^{(x-3)}$$

$$\log a^{x+2} = \log b^{x-3}$$
$$(x+2)\log a = (x-3)\log b$$
$$x \log a + 2\log a = x \log b - 3 \log b$$
$$2 \log a + 3 \log b = x \log b - x \log a$$
$$2 \log a + 3 \log b = x(\log b - \log a)$$
$$\frac{(2 \log a + 3 \log b)}{(\log b - \log a)} = x$$