## RATE OF CHANGE PROBLEMS.

$$
\text { KEY IDEA: The "rate of increase of } B \text { " is } \frac{d B}{d t}
$$

1. If $\boldsymbol{B}=5 \boldsymbol{t}-\mathbf{6}$ find the rate of increase of B
2. If $\boldsymbol{P}=\boldsymbol{t}^{2}+3 \boldsymbol{t}$ find the rate of increase of P when $\mathrm{t}=4$
3. If $V=\boldsymbol{t}^{3}-\boldsymbol{\sigma} \boldsymbol{t}$ find the rate at which $V$ is changing when:
(i) $t=10$
(ii) $t=1$

NB If the rate of change of $V$ is POSITIVE then $V$ is INCREASING.
If the rate of change of $V$ is NEGATIVE then V is DECREASING.
4.(a) Suppose a drop of water falls on a paper towel and the circular wet patch increases in radius according to the equation $r=2 t+3$ where $t$ is in seconds and $r$ is the radius in $\mathbf{m m}$.

Find:
(i) The rate of increase of the radius in $\mathrm{mm} / \mathrm{sec}$
(ii) The rate of increase of the Circumference

$$
\begin{aligned}
C & =2 \pi r \\
& =2 \pi(2 t+3) \\
& =
\end{aligned}
$$

Rate of Inc $=\frac{d C}{d t}=\quad \mathrm{mm} / \mathrm{sec}$
(iii) The rate of increase of the AREA of the wet patch at $\mathrm{t}=1 \mathrm{sec}$ :

$$
\begin{aligned}
A & =\pi r^{2} \\
& =\pi(2 t+3)^{2} \\
& =\pi\left(4 t^{2}+12 t+9\right) \\
& =
\end{aligned}
$$

Rate of increase is:

$$
\frac{d A}{d t}=
$$

5. Suppose a model car moves so that its distance, $s$, from O , at t sec is:

$$
s=t^{2}+3 t+2 \text { metres }
$$

The "rate" at which the distance increases is what we call the "velocity" so we can say, velocity $\boldsymbol{v}=\frac{d s}{d t} \quad$ metres per second or $\mathrm{m} / \mathrm{s}$ or $\mathrm{ms}^{-1}$
The "rate" at which the velocity increases is what we call the "acceleration" so we can say, acceleration $a=\frac{d V}{d t} \quad m / s$ every sec or $m / s / s$ or $\mathrm{ms}^{-2}$

If $s=t^{2}+3 t+2$ find the values of $s, v$ and a at $t=4 \mathrm{sec}$
(i) $s=$
(ii) $v=$
(iii) $a=$
6. A ball is kicked vertically up into the air so that its height, h metres, at $t \sec$ is : $\quad \boldsymbol{h}=\mathbf{3 0 t}-\mathbf{5} \boldsymbol{t}^{2}$
(a) Find the velocity equation: $V=\frac{d \boldsymbol{h}}{\boldsymbol{d} \boldsymbol{t}}$
(b) Find the acceleration equation : $\boldsymbol{a}=\frac{d V}{d t}$
(c) Find the time when $\boldsymbol{V}=\mathbf{0}$
(d) Find the greatest height reached by the ball.
(e) Find the two times when $\boldsymbol{h}=\boldsymbol{0}$
(f) Find the two times when $\boldsymbol{h}=20$ metres.
(solve the quadratic equation using graphics calculator)
7. A stone is thrown vertically up from the top of a cliff which is 25 metres high. The height of the stone from the top of the cliff at t secs is:

$$
h=20 t-5 t^{2}
$$

Find (a) the velocity equation
(b) the acceleration equation
(c) the initial velocity of the stone
 ("initial" means at $\mathrm{t}=0$ )
(d) the time when the velocity is zero
(e) the maximum height the stone reaches
(f) the time taken for the stone to reach the bottom of the cliff.
(d) the speed that the stone hits the beach at the bottom of the cliff.
8. A model car moves in a straight line and its distance from the starting point is given by : $\boldsymbol{x}=\boldsymbol{t}(\boldsymbol{t}-\boldsymbol{6})^{2}$ The motion lasts from $\boldsymbol{t}=\boldsymbol{0}$ to $\boldsymbol{t}=\boldsymbol{6} \mathrm{sec}$
( The graph would be a simple cubic curve.

The velocity will be a simple parabola

$$
\text { If } \begin{aligned}
x & =t(t-6)^{2} \\
& =t\left(t^{2}-12 t+36\right) \\
& =t^{3}-12 t^{2}+36 t
\end{aligned}
$$

$$
\begin{aligned}
v & =3 t^{2}-24 t+36 \\
& =3\left(t^{2}-8 t+12\right) \\
& =3(t-2)(t-6)
\end{aligned}
$$

and the acceleration will be a line graph. accel, $\boldsymbol{a}=\boldsymbol{\sigma} \boldsymbol{t}-\mathbf{2 4}$
The velocity will be zero when

$$
\begin{aligned}
3 t^{2}-24 t+36 & =0 \\
\text { ie } 3( & ) \\
\text { ie } 3(\quad)() & =0 \\
\text { so } t= &
\end{aligned}
$$

Max distance from O will be when $t=$

## Max dist $=$



This car start from O with an initial velocity of $36 \mathrm{~m} / \mathrm{s}$ and is slowing down until the speed becomes momentarily zero at $\mathrm{t}=2 \mathrm{sec}$, a maximum distance of 32 m from O . The car then goes backwards reaching a max speed of $12 \mathrm{~m} / \mathrm{s}$ at $\mathrm{t}=4 \mathrm{sec}$ (actually, a min Velocity of -12 ) It then slows down reaching O , stopping at $\mathrm{t}=6$.
9. A "remote controlled car" sets off from the base of a wall at W and accelerates up to a certain maximum speed. It then decelerates until it momentarily comes to test and then accelerates in reverse along the same path, finally crashing into the wall at its original starting point W .
The equation for S , the distance from the wall in metres, at t seconds is :

$$
S=t^{2}(9-t) \quad \text { for } \quad 0 \leq t \leq 9
$$

(a) Find the velocity equation.
(i) Find the initial velocity
(ii) Find the time at which $\boldsymbol{v}=\mathbf{0}$
(iii) Find the car's maximum distance from the wall.
(b) Find the acceleration equation.

(i) Find the initial acceleration.
(ii) Find the time at which the acceleration is zero.
(iii) Find the car's maximum velocity.
(iv) How far from the wall is the car when it stops accelerating and starts decelerating. (ie when $\boldsymbol{a}=\boldsymbol{0}$ )
(c) At what time does the car hit the wall?
(d) At what speed does the car hit the wall ?
(e) Draw careful graphs of distance, velocity and acceleration of the car for $\boldsymbol{t}=\mathbf{0}$ to 9 sec .


