MERIT QUESTIONS ON A TYPICAL NCEA PAPER. (B)

ALGEBRA

Question ONE

(a) Calculate x if $\log_4 18 = x$

$4^x = 18$

$\log 4^x = \log 18$

$x \log 4 = \log 18$

$x = \dfrac{\log 18}{\log 4}$

$= 2.08$

(b) Solve for x $\quad 3^x = 2^{x+1}$

$\log 3^x = \log 2^{x+1}$

$x \log 3 = (x+1) \log 2$

$x \log 3 - x \log 2 = \log 2$

$x(\log 3 - \log 2) = \log 2$

$x = \dfrac{\log 2}{\log 3 - \log 2}$

$= 1.71$

Question TWO

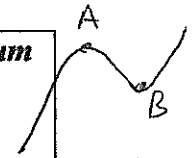**(a) Solve $(3^x)^2 - 7 \times 3^x - 18 = 0$**

$(3^x + 2)(3^x - 9) = 0$

$3^x = -2$ (not possible) or $3^x = 9$

So $x = 2$

(b) Solve

$\dfrac{x^2 - 9}{x - 3} = 5$

$\dfrac{(x-3)(x+3)}{(x-3)} = 5$
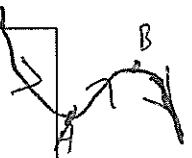
$x \neq 3$

$x + 3 = 5$

$x = 2$

Question THREE

(a) Find p if the equation $x^2 + (3p+2)x + (5p+6) = 0$ only has one root.

$\Delta = 0$

$(3p+2)^2 - 4(5p+6) = 0$

$9p^2 + 12p + 4 - 20p - 24 = 0$

$9p^2 - 8p - 20 = 0$

$p = 2$ or -1.11

(b) The equation $x^2 + 2kx + (k+2) = 0$ has no real solutions. Find the possible k values.

$\Delta < 0$

$4k^2 - 4(k+2) < 0$

$4k^2 - 4k - 8 < 0$

$k^2 - k - 2 < 0$

$(k-2)(k+1) < 0$

$-1 < k < 2$

CALCULUS

Question ONE

(a) The velocity of a ball kicked vertically up is $v = 20 - 10t$. If the ball was kicked from an initial height of 1 metre, find the equation for its height at t sec.

$x = 20t - 5t^2 + c$

Sub $t = 0$, $x = 1$ so $c = 1$

$x = 20t - 5t^2 + 1$

(b) Find the x and y values of the maximum and minimum points on the curve $y = x^3 - 12x^2 + 27x + 2$. (state the nature of the turning points)

$y' = 3x^2 - 24x + 27 = 0$ at A and B

$3(x^2 - 8x + 9) = 0$

$x = 1.35$ and 6.65

$\text{Max}(1.35, 19)$ $\quad$ $\text{Min}(6.65, -55)$

Question TWO

(a) Find the equation of the tangent to the curve $y = x^2 + 3x$ at the point where $x = -1$

$y = 1 - 3 = -2$

$y' = 2x + 3$

$= 1$ ← grad

$\tan \Rightarrow y = mx + c$

$-2 = 1 \cdot -1 + c$

$-1 = c$

$y = x - 1$

(b) For what values of x is the curve $y = 6x^2 - x^3$ a decreasing function?

$y' = 12x - 3x^2 = 0$ at A and B

$3x(4 - x) = 0$

$x = 0, 4$

decreasing if $x < 0$ and $x > 4$

Question THREE

(a) A model rocket starts off at ground level with an initial velocity of 1 m/s and with an acceleration of $\dfrac{dv}{dt} = 12t$. Find the equation for the height of the rocket at t secs.

$v = 6t^2 + c$

Sub $t = 0$, $v = 1$ so $c = 1$

$v = 6t^2 + 1$

$h = 2t^3 + t + d$

$h = 0$, $t = 0$, so $d = 0$

$h = 2t^3 + t$

(b) Find the equation of a curve if its gradient is $y' = x^2 + x$ and it passes through the point (6, 0)

$y = \dfrac{x^3}{3} + \dfrac{x^2}{2} + c$

Sub $x = 6$, $y = 0$

$0 = 90 + c$

$y = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 90$