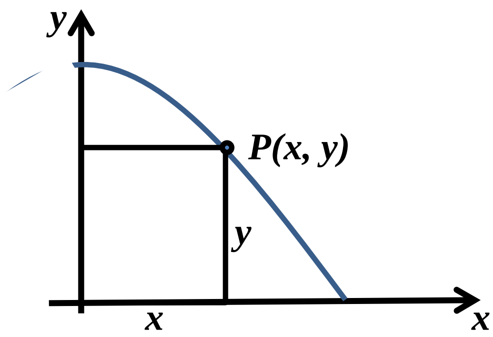
ACADEMIC TYPE MAX/MIN PROBLEMS.

1. The graph shown is $y = 12 - x^2$
for $0 \le x \le \sqrt{12}$
P is the general point (x, y) on the curve

A rectangle is drawn passing through P and the origin (0, 0)

Find the maximum area of the rectangle.



The Area of the rectangle is $A = xy$

Subs $y = 12 - x^2$ and we get $A = x(12 - x^2)$

$A = 12x - x^3$

Differentiating $A' = 12 - 3x^2 = 0$ ***for max area***

$3x^2 = 12$

$x^2 = 4$

$x = 2$ *since* $0 \le x \le \sqrt{12}$

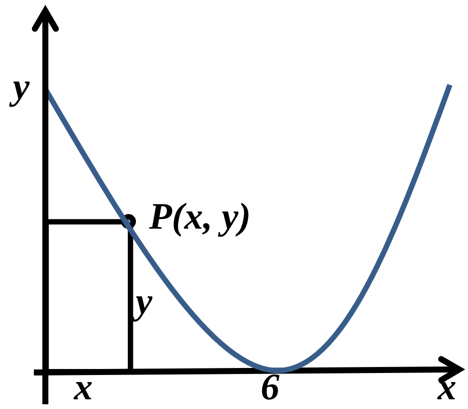
If $x = 2, y = 12 - 4 = 8$

So max area $= 16$ ***cm***^{2}

2. The graph shown is $y = (x - 6)^2$
for $0 \le x \le 6$
P is the general point (x, y) on the curve

A rectangle is drawn passing through P and the origin (0, 0)

Find the maximum area of the rectangle.



The Area of the rectangle is $A = xy$

Subs $y = (x - 6)^2$ and we get $A = x(x - 6)^2$

$A = x(x^2 - 12x + 36)$

$A = x^3 - 12x^2 + 36x$

Differentiating $A' = 3x^2 - 24x + 36 = 0$ ***for max area***

$3(x^2 - 8x + 12) = 0$

$3(x - 2)(x - 6) = 0$

$x = 2$ ***or*** $x = 6$

If $x = 6, y = 0$ ***so the area*** $= 0$ ***. This must be the Min area.***

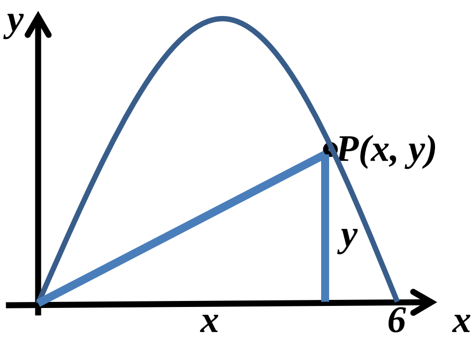
Max area is when $x = 2, y = 16$

Max area $= 32$ ***cm***^{2}

3. The graph shown is $y = 6x - x^2$
for $0 \le x \le 6$
P is the general point (x, y) on the curve

A triangle is drawn passing through P and the origin (0, 0)

Find the maximum area of the triangle.



The Area of the triangle is $A = \frac{xy}{2}$

Subs $y = 6x - x^2$ and we get $A = \frac{x(6x - x^2)}{2}$ so $A = \frac{6x^2 - x^3}{2}$

Differentiating $A' = \frac{12x - 3x^2}{2} = 0$ for max area

$$12x - 3x^2 = 0$$
$$3x(4 - x) = 0$$
$$x = 4 \text{ or } 0$$

If $x = 0$, $y = 0$ so Area = 0. This must be the MIN area.

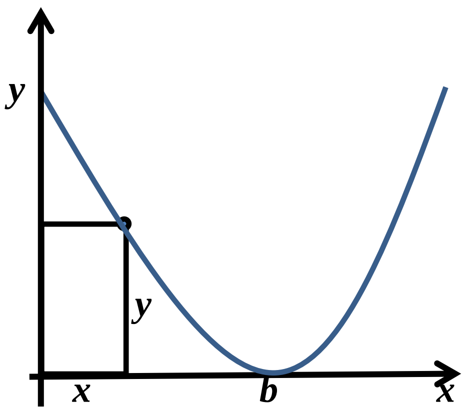
If $x = 4$, $y = 24 - 16 = 8$

So max area = $\frac{4 \times 8}{2} = 16\ cm^2$

If $x = 2$, $y = 12 - 4 = 8$

So max area = $16\ cm^2$

4. The graph shown is $y = (x - b)^2$
for $0 \le x \le b$
P is the general point (x, y) on the curve
A rectangle is drawn passing through P and the origin (0, 0)
Find the maximum area of the rec.angle.



The Area of the rectangle is $A = xy$

Subs $y = (x - 6)^2$ and we get $A = x(x - b)^2$

$$A = x(x^2 - 2bx + b^2) = x^3 - 2bx^2 + b^2x$$

Differentiating $A' = 3x^2 - 4bx + b^2 = 0$ for max area

$$(3x - b)(x - b) = 0$$
$$x = \frac{b}{3} \text{ or } x = b$$

If $x = b$, $y = 0$ so the area = 0. This must be the Min area.

Max area is when $x = \frac{b}{3}$, $y = \left(\frac{b}{3} - b\right)^2 = \frac{4b^2}{9}$

Max area = $\frac{b}{3} \times \frac{4b^2}{9} = \frac{4b^3}{27}\ cm^2$